PCTM Magazine

WHAT DO YOU SEE?

An Official Publication of the Pennsylvania Council of Teachers of Mathematics

FRINGE THOUGHT: A hero is no braver than an ordinary man. He's just brave 5 minutes longer.

The sculpture on the cover is one of the "Dolphins on Parade" scattered around Atlanta, GA. The sculptures promote the dolphin exhibit in the Georgia Aquarium.

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PCTM Magazine is published three times each year by the Pennsylvania Council of Teachers of Mathematics, an affiliate of the National Council of Teachers of Mathematics.

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The **PCTM Magazine** editors are:
Cynthia Taylor and Tyrone Washington (Millersville University)

President's Message

Lynn Columba

Dear PCTM Members:

*No matter how lucidly and patiently teachers explain **to** their students, they cannot understand **for** their students.*

Schifter & Fosnot (1993, p. 9)

I would like to encourage you to reflect on the above quote and your philosophy of teaching mathematics. To begin your reflections, consider the answer to the following questions:

- What type of environment creates an enthusiasm for learning mathematics for the students in your classroom?
- What is your "best" lesson or the lesson in which all the students were learning/understanding and achieving in the mathematics classroom?
- What do you want to accomplish with your teaching? What are the goals for your students' learning mathematics?
- What do you value? What is important to you?
- What research-affirmed teaching strategies or approaches produce the best learning environment for mathematics instruction?
- What does research say about the mathematics teaching strategies that you have selected for your classroom?

I believe that ***all*** children can learn and that each child learns in his/her own unique way. The environment that best nurtures this learning provides concrete, hands-on experiences so learners can construct their own understandings. In addition, discourse (verbal descriptions) and reflection enhance the learning process. Seymour Papert, a student of Piaget's and an expert on how technology can provide new ways to learn, said, "Children learn by doing and by thinking about what they do." These interactive, manipulative explorations occur in large and small groups, which can be peer- or teacher-led. Instructional variety and students' active participation are the key elements. Problem-solving methodology in the classroom allows students to confront situations in which they do not immediately know the answer. Using problem-solving strategies, such as guess-and-check or working backwards, help students develop life-long thinking skills. This "thinking curriculum" prepares students for real-life situations in the post-school world they will be entering.

My philosophy of teaching mathematics focuses on the development of concepts. Instruction begins at the conceptual level where students are exploring new ideas with manipulatives and engaging in dialogue in small and large groups. At the connecting level, students are linking symbols to the manipulatives and classroom discussions and modeling. The symbolic level can be viewed as a written record of what the students have been doing with the manipulatives and the dialogue with their classmates and the teacher. See Figure 1 for a visual model of my philosophy of teaching mathematics.

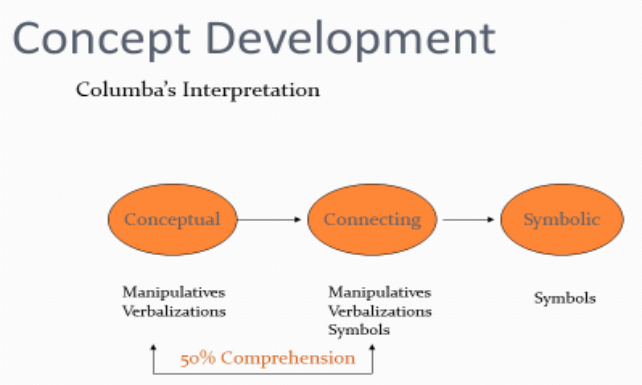


Figure 1. Columba's philosophy of teaching mathematics.

Another conceptually oriented instructional model is Concrete-Representational-Abstract (CRA), a three-step instructional approach that has been found to be highly effective in teaching mathematical concepts (Steedly, Dragoo, Arefeh, & Luke, 2008). The first step, the concrete stage, known as the "doing" stage, involves physically manipulating objects to solve a mathematics problem. The representational (semi-concrete) stage is the "seeing" stage and involves using images to represent objects to solve a math problem. The final step is the abstract or symbolic stage, which is known as the "symbolic" stage and involves using only numbers and symbols to solve a math problem. CRA is a gradual systematic approach where each stage develops from the previous stage and, therefore, must be taught sequentially. This approach can be found at all levels, from elementary through high school classrooms and in special needs classrooms (Flores, 2009; Manci, Miller, & Kennedy, 2012; Strickland & Maccini, 2013).

A classroom mathematics teacher is to guide students as they build their bridge to abstract learning. Often this bridge looks different for different students when we differentiate to meet unique instructional needs. All students need to explore with manipulatives and to engage in purposeful, intentional dialogue to develop conceptual understanding. If students do not have an opportunity to work at the conceptual and representational levels, then they are often left to memorizing instead of developing understanding.

FRINGE THOUGHT: The average person was born in the country, worked hard to live in the city, then worked even harder to get back to the country.

The frequent review and reflection on your philosophy of teaching mathematics is vital because it affects instructional decisions, including planning, teaching, and assessing. Our goal to raise achievement in mathematics for every student requires that teachers understand which teaching approaches best fit the content and how to best organize the content elements for effective teaching. Pedagogical content knowledge is the critical knowledge that connects specific mathematics content with effective mathematics instruction. It represents a blend of what content to teach and how best to teach it (NCSM, 2014).

I hope you enjoyed your Winter Break. As you prepare for a new year, imagine a classroom or school district where *all* students:

1. Participate in high quality mathematics learning experiences;
2. Achieve at high levels and grow academically;
3. Engage in mathematics that is meaningful, relevant, and accessible;
4. Graduate college- or career -ready; and
5. Can contribute to society as mathematically literate citizens.

Sincerely,
Lynn Columba
PCTM President

Flores, M. (2009). Teaching subtraction with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure*, 53, 145-152.

Manci, D. B., Miller, S.P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrate strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice, 27,*152-166. Doi:10.1111/j.1540-586.2012.00363.x

National Council of Supervisors of Mathematics. (2014). *It's TIME: Themes and imperatives for mathematics education: A leadership framework for Common Core Mathematics.* Bloomington, IN: Solution Tree and National Council of Supervisors of Mathematics.

Schifter, D., & Fosnot, C. T. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform.* New York: Teachers College Press.

Steedly, K., Dragoo, K., Arefeh, S., & Luke, S. D. (2008). Effective mathematics instruction. *Evidence for Education*, 3 (1), 1–12. Retrieved from http://nichcy.org.research/ee/math

Strickland, T. K., & Maccini, P. (2013). The effects of the concrete–representational–abstract integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education*, 34(3), 142–153. doi:10.1177/0741932512441712

FRINGE THOUGHT: Any good idea will work eventually—if you do.

Call for Speakers for the 2017 PCTM Annual Conference

PCTM is pleased to announce the Call for Proposals for the 66th Annual Conference to be held **August 2-3, 2017** at Hilton Harrisburg in Harrisburg, PA.

Suggested Topics and Session Information

Sessions are intended to actively engage participants: We surveyed past speakers and participants for topics that they find to be important and of interest. Please consider this list as inspiration for your presentation:

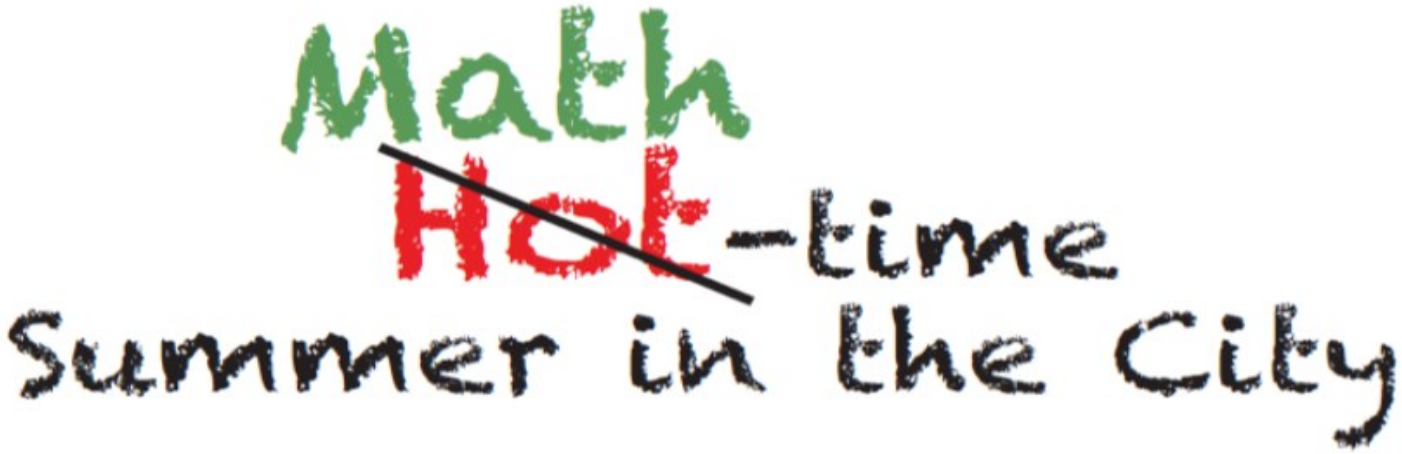
- Ways of Improving PSSA and Keystone Test scores by focusing on providing details to support open-ended questions
- Understanding teacher/student PVAAS scores and how they can be used to improve teaching
- Share practical ideas for reaching English Language Learners, Economically Disadvantaged, IEP and other At-Risk students in the math classroom (PreK - 12)
- Exploring successful ways for motivating students in the math classroom
- Teaching students to pay attention to details and attend to precision when problem solving
- Using and integrating technology in innovative ways into the mathematics classroom
- Developing number sense and fluency in Early Childhood Mathematics Education

Presentation Format

- Bursts (20 minutes) Bursts are short, concise presentations that focus on a specific topic or idea. The goal of these presentations is information sharing, conveyed quickly and succinctly.
- Session (45 minutes) Sessions allow speakers to convey information about multiple topics or broad ideas in lecture format.
- Workshop (75 minutes) Workshops provide attendees the opportunity to work with hands-on activities in an interactive environment setting.

Submit your proposal by **February 1, 2017** at http://www.pctm.org/

FRINGE THOUGHT: Desk: A wastebasket with drawers.



NCTM Regional Conference in Philadelphia

NCTM Regional Conference & Exposition 2016
PHILADELPHIA | OCT 31-NOV 2

The NCTM Regional Conference in Philadelphia on November 1st and 2nd would not have been possible without the generous help of our many volunteers.
In appreciation, PCTM teamed up with ATMOPAV to award an IPad to a randomly chosen volunteer. The lucky winner was Heather Porter, a student from Susquehanna University. She is pictured below with fellow students Maggie Crann and Lauren Creamer. Good luck to these future math teachers!

Left to Right in the Photo: Maggie Crann, Lauren Creamer, and Heather Porter

FRINGE THOUGHT: Old proverb: Drive thy business, let it not drive thee.

Seated at the PCTM table, in the Convention Center in Philadelphia for the NCTM 2016 Regional Conference November 1-2, 2016, (from left to right) Ryan Mulville and Sarah Hafer, both PCTM Interns and Temple sophomores. Not pictured are Lynn Columba, PCTM President, and Amanda Schantz, PCTM Intern Program Coordinator.

Call for Nominations for the PCTM Board

Dear PCTM Members,

The Nominations and Elections Committee is soliciting nominations for the PCTM Board. The term of office runs from July 1, 2017 to June 30, 2019. The election will take place in early March 2017. The slate of candidates will be approved at the February 25, 2017 Board meeting. Please self-nominate or nominate a colleague (with their permission). Also, please submit a brief bio and a photo with the nomination to Marian Avery at mavery077@gmail.com. Nominations are due by **February 3, 2017.**

There are openings for the following position on the PCTM Board, which are all two-year terms:

- **Delegate at Large**
- **Eastern Regional Representative**
- **Central Regional Representative**
- **Western Regional Representative**

The Delegate at Large position is for any mathematics based educator employed in the state of Pennsylvania, including the classroom at any level or Intermediate Unit. The Regional Representative positions are for K-12 classroom teachers, one each from the eastern, central, and western part of Pennsylvania.

Those elected to these positions will be invited to attend the May 20, 2017 Board meeting. Expectations of the office include attendance at the four Board meetings held each fiscal year (one of which is held at the annual conference) and helping with the PCTM Annual Conference. This position offers the opportunity to be involved more deeply with the workings of PCTM and the Annual Conference, and a voice to share ideas, suggestions, and thoughts at Board meetings. People serving in these positions are encouraged to solicit the thoughts of their colleagues regarding mathematics education/teaching in Pennsylvania and share them with the Board as to how PCTM may better serve the teachers of Pennsylvania.

Please contact me with any questions at mavery077@gmail.com and we look forward to your enthusiastic participation in this year's election!

Sincerely,
Marian E. Avery, Chair
Mary Lou Metz
Dave Frederickson

FRINGE THOUGHT: Coincide: What you do when it's raining.

GREAT MATH TEACHING IDEAS

What are you doing with your students?

Send a picture and a short description to pctm.editor@gmail.com

We need your help filling this space!

PCTM Awards

PCTM would like to invite you to consider nominating a colleague for one of our annual awards. These awards are intended to recognize and encourage mathematical excellence in our profession. If you have the pleasure of working with a colleague who meets or exceeds the qualifications for any of the awards listed below, please take a few minutes to write a short, clear synopsis of his/her qualifications to share with the nominating committee. Through the awards, PCTM seeks to foster, develop, and spread enthusiasm for teaching mathematics across Pennsylvania within its membership. We also want to acknowledge service to the profession from those dedicated members who actively volunteer their time on behalf of our organization. During this busy time, we sincerely hope you will take the time to nominate a worthy candidate for this year's PCTM awards.

Deadline for Annalee Henderson is March 15th.
Deadline for all other awards (given in summer) is June 1st.
Send nominations to: Jo Kinsey (JKinsey@dasd.org) or Scott Bennetch (scott_bennetch@cocalico.org).

The Annalee Henderson Outstanding Student Award
This award recognizes graduating high school seniors who have exhibited excellence in mathematics. Those nominating a senior for the award must submit in writing to the Awards Committee, a summary of the candidate's achievements. The Awards Committee will submit to the Executive Board the name of the student who is recommended to receive this award. The Annalee Henderson Award is given in May.
Deadline: March 15th

Outstanding Contribution to PCTM Award
This award will be presented to one of the select few members of the Council who, over the years, has demonstrated outstanding service and leadership to the organization. The individual nominating an educator for this award must submit in writing to the Awards Committee, a summary of the candidate's contributions. The Awards Committee will submit to the Executive Board the name of the educator who is recommended to receive the award.
Deadline: June 1st

The Outstanding Contribution to Mathematics Education Award
This award will be presented to a mathematician, or a mathematics educator from a public or private school, college or university, or industry, who has made an outstanding contribution to mathematics education in Pennsylvania. The individual nominating a candidate for this award must submit in writing to the Awards Committee, a summary of the candidate's contributions. The Awards Committee will submit to the Executive Board the name of the educator who is recommended to receive the award.
Deadline: June 1st

The PCTM New Teacher Award
The purpose of the award is to recognize promising, beginning teachers of mathematics so that they will be encouraged to remain in the profession. To be nominated for this award a candidate must:

- Be a teacher in a public/private school (at any level, K-12) who teaches mathematics.
- Have completed his/her first, second, or third year of his/her first full-time teaching experience.
- Be nominated by a supervisor, an administrator, or a colleague.
- Exhibit contagious enthusiasm for students' learning of mathematics.
- Demonstrate initiatives in developing innovative strategies in his/her teaching.
- The individual nominating a candidate for this award must submit in writing to the Awards Committee, a summary of the candidate's contributions. The Awards Committee will submit to the Executive Board the name of the educator who is recommended to receive the award.

The award, consisting of a plaque and three years of free membership of PCTM, will be presented at the Annual PCTM Conference. The individual recommending a candidate for this award must submit a summary of the candidate's contributions towards the teaching of mathematics to the Awards committee.
Deadline: June 1st

FRINGE THOUGHT: If you wake up and find success, you haven't been asleep.

The Mathematics Hall of Fame Award

This award will be presented to the mathematics educator in Pennsylvania who:

- is regarded by his/her peers as making the greatest impact on mathematics students and
- has continually exemplified excellence as a mathematics educator.

The individual nominating an educator for this award must submit in writing to the Awards Committee, a summary of the candidate's contributions. The Awards Committee will submit to the Executive Board the name of the educator who is recommended to receive the award.

Deadline: June 1st

The PCTM Masters of Mathematics Award

This award will recognize math educators at four different levels of education:
Primary Master of Mathematics: Kindergarten – Grade 2
Intermediate Master of Mathematics: Grades 3-5
Middle School/Junior High School Master of Mathematics
High School Master of Mathematics
To be nominated for this award a candidate must:

1. Be a teacher in a public/private school (at any level, K-12) who teaches mathematics.
2. Have completed at *least* seven years of full-time teaching
3. Be nominated by a supervisor, an administrator, or a colleague.
4. Exhibit contagious enthusiasm for students' learning of mathematics.
5. Demonstrate initiatives in developing innovative strategies in his/her teaching.

The individual nominating a candidate for this award must submit in writing to the Awards Committee, a summary of the candidate's contributions. The Awards Committee will submit to the Executive Board the name of the educator who is recommended to receive the award.

Deadline: June 1st

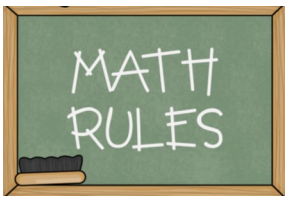
New Teacher Award

John Tyler Garey is the 2016 recipient of the New Teacher Award for the Pennsylvania Council of Teacher of Mathematics (PCTM). John has completed his first year of teaching at Penn Wood High School in Yeadon, PA. He has taught Algebra I at all levels from Keystone Remediation through Honors and Geometry and was nominated by Daniel Wisniewski, Head of the Division of Sciences and Mathematics at DeSales University in Center Valley, PA. Wisniewski taught and mentored Garey during his undergraduate years and now collaborates with him on mathematics education projects.

Dr. Judy Lee, Assistant Principal at Penn Wood School also nominated John Garey for the New Teacher Award. Dr. Lee noted that John brings great enthusiasm to his work and collaborates easily with his fellow teachers, notably on the technology committee and the steering committee for Penn Wood's new 9th Grade Academy. Dr. Lee has also stated, "John brings his whole self to teaching. Because he is a genuine, open, and humble person, his students feel safe enough to try to meet the challenges with which he presents them." To quote Dr. Lee further, "John strives to make the material he teaches relevant to the lives of his students. . . His class is a lively classroom in which problem solving and critical thinking are always promoted. It is obvious to his colleagues and students that he is attempting to build in them a passion for learning that will energize them for college."

The PCTM New Teacher Award is given to math educators who have taught three years or less in a school in Pennsylvania. PCTM seeks to recognize educators who have taught with contagious enthusiasm and skill during their teaching years.

John Garey, right, the New Teacher awardee for 2016 and his nominator Brother Dan Wisniewski, left. John teaches math at Penn Wood High School in Yeadon, PA.



FRINGE THOUGHT: He who laughs…lasts.

Seeing Stars with Random Sampling

Robert Lochel

Online applets tailored for statistical investigations allow students to experience new ideas through simulation. A recent activity I used in my AP Statistics classes, adapted from Introduction to Statistical Investigations, AP Version, by Tintle, Chance, Cobb, Rossman, Roy, Swanson and VanderStoep, helped my students understand the differences between simple, stratified and cluster samples. Ruth Carver opened my eyes to this approach at a recent meeting of the Philadelphia-Area Statistics Teachers Association (PASTA), and her enthusiasm and willingness to share are always appreciated. This particular activity uses the one-variable sampling applet from the Rossman-Chance applet collection, and is ideal for 1-1 classrooms, or even students working in tech teams.

In "Sampling Stars," students use simulation to estimate the number of stars in a hypothetical sky. Note, the start of this activity does not seem much different from "Random Rectangles" or "Jelly Blubbers" – two activities often used in AP Statistics to model random sampling – except that this "population" seems much less countable. First, students encounter the "sky," which has been broken into 100 squares (see Figure 1).

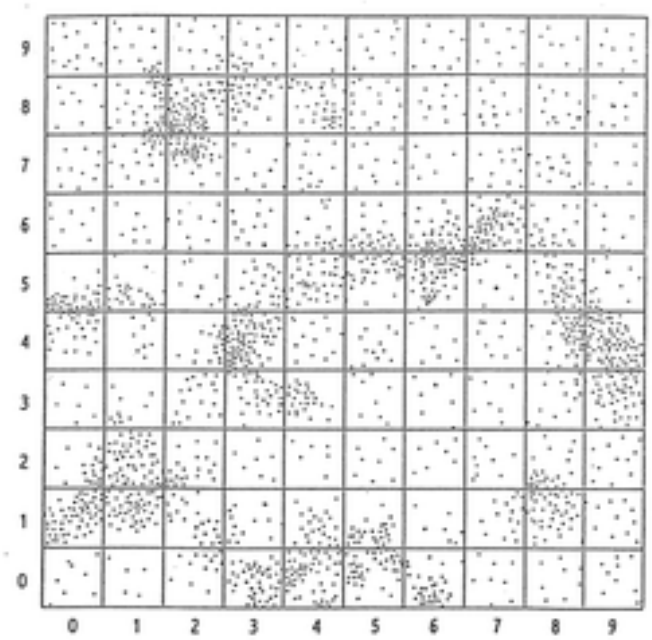


Figure 1. The sky.

To start, teams work to define procedures for selecting a random sample of 10 squares, using both the "hat" (non-technology) method, and a method using technology (usually a graphing calculator). Before we draw the samples however, I want students to think about the population. Specifically, will a random sample do a "good job" with providing estimates? Groups were asked to discuss what they notice about the sky. My classes provided many noteworthy observations, but all eventually agreed on one observation which could influence our estimates: "There are some squares where there are many stars (we wound up calling these "dense" squares) and some where there are not so many."

Before we drew our first sample, we talked about the need to consider both dense and non-dense areas in our sample, and the possibility that our sample will overestimate or underestimate the population, even in random sampling. There are a lot of stats goodness in all of this, and the conversation felt natural and accessible to the students.

Students then used their technology-based procedure to draw a random sample of 10 squares, marking off the squares. But counting the actual stars is not reasonable, given their quantity - so it's Beth Chance to the rescue! Make sure you click the "stars" population in the applet to get started. Beth has provided the number of stars in each square, and information regarding density, row and column to think about later (see Figure 2).

FRINGE THOUGHT: The mark of a motivated man is his ability to distinguish a setback from a defeat.

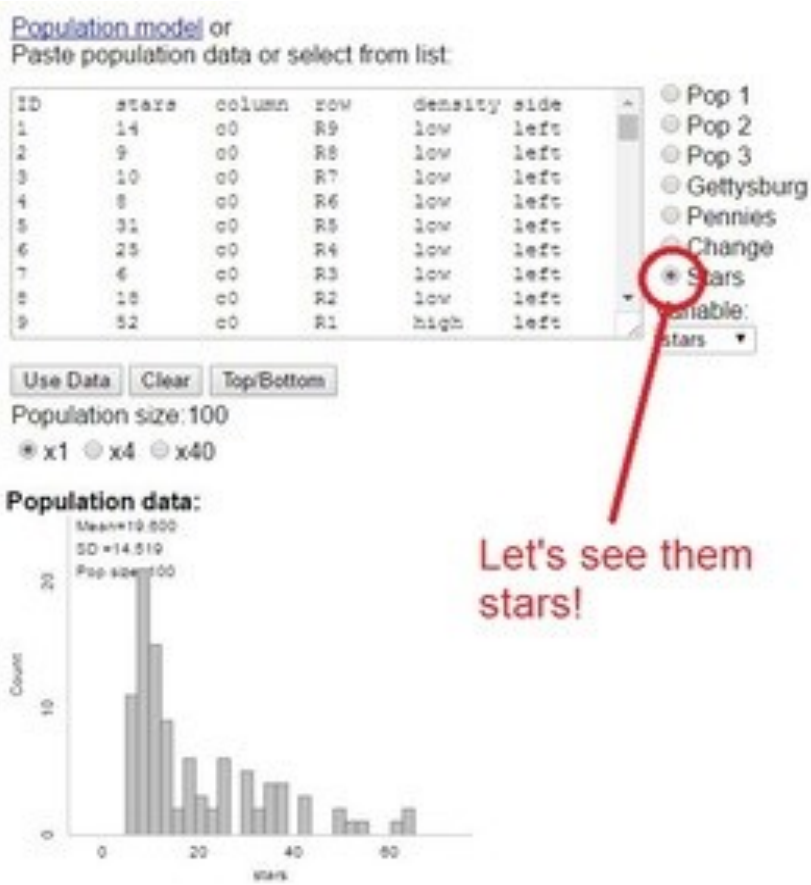


Figure 2. Sample graph.

Before we start clicking blindly, let's describe that population. The class quickly agrees that we have a skewed-right distribution, and takes note of the population mean - we'll need it to discuss bias later.

Clicking "show sampling options" on the top of the screen allows us to simulate random samples. First, students each drew a sample of size 10 - the bottom of the screen shows the sample, summary statistics, and a visual of the 10 squares chosen from the population (see Figure 3).

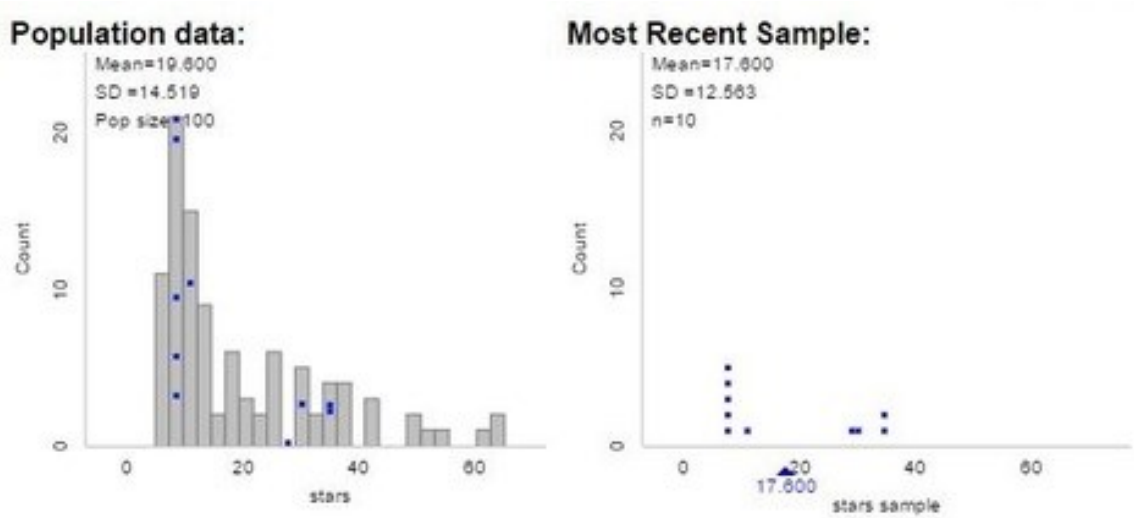


Figure 3. Sample size of 10.

Groups were asked to look at their sample means, share them with neighbors, and think about how close these samples generally come to hitting their target. Find a neighbor where few "dense" area were selected, or where many "dense" squares made the cut, how much confidence do we have in using this procedure to estimate the population mean?

FRINGE THOUGHT: The largest room in the world is the room for improvement.

Eventually I unleashed the sampling power of the applet and let students draw more and more samples. And while a formal discussion of sampling distributions is a few chapters away, we can make observations about the distributions of these sample means. See Figure 4.

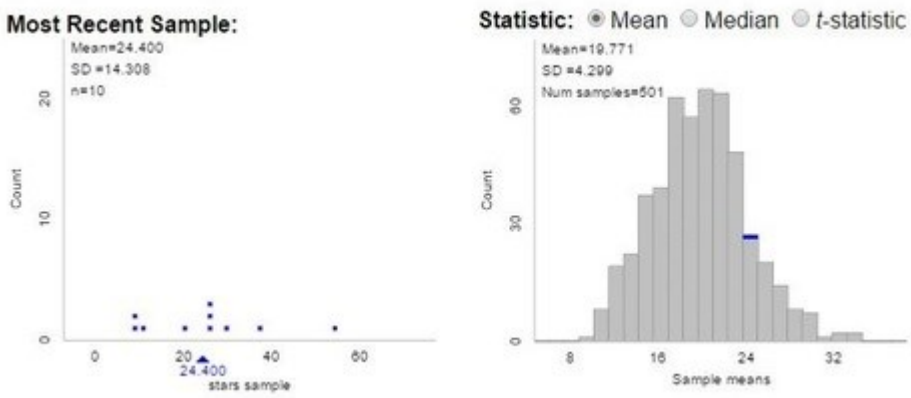


Figure 4. Distributions of sample means.

I knew the discussion was heading in the right direction when a student observed, "Hey, the population is definitely skewed, but the means are approximately normal. That's odd..."

Yep, it sure is...and more seeds have been planted for later sampling distribution discussions. But what about those dense and non-dense areas the students noticed earlier? Our random samples seem to provide an unbiased estimator of the population mean, but can we do better? This is where Beth's applet is so wonderful, and where this activity separates itself from Random Rectangles. On the top of the applet, we can stratify our sample by density, ensuring that an appropriate ratio of dense / non-dense areas (here, 20%) is maintained in the sample. The applet then uses color to make this distinction clear. Here, green dots represent dense-area squares. See Figure 5.

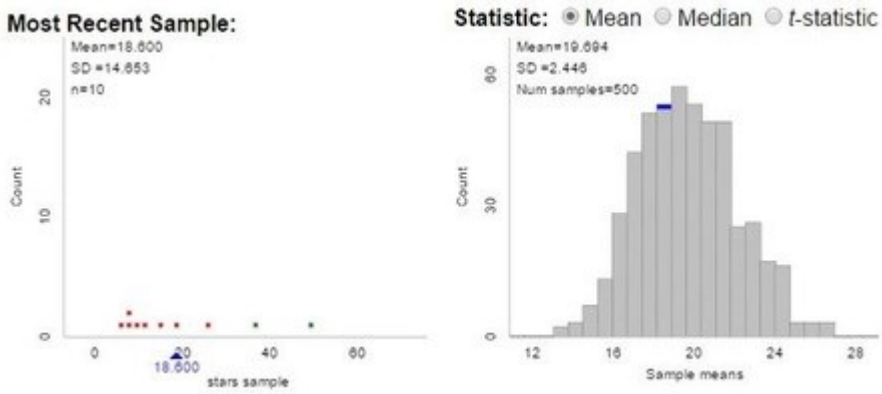


Figure 5. Stratified sample.

Finally, note the reduced variability in the distribution from stratified samples, as opposed to random samples. The payoff is here!

In a future article, we will look at samples stratified by row and/or column. Cluster samples by row or column will also make an appearance. There is so much to talk about with this one activity, and I appreciate Ruth and Beth for sharing! Consider the power of simulation and free online applets in your statistics courses.

Robert Lochel is a teacher at Hatboro-Horsham High School, and an AP Statistics reader. He is the president of ATMOPAV, and was recently named to the first Desmos Fellowship group.

The Pennsylvania Mathematics Education Coalition

Kate Remillard

In Summer 2016, the leadership of Pennsylvania's three prominent mathematics education professional organizations (Pennsylvania Council of Teachers of Mathematics [PCTM], Pennsylvania Association of Mathematics Teacher Educators [PAMTE], and Pennsylvania Council of Leaders of Mathematics [PCLM]) convened to discuss the potential for collaboration in the area of advocacy. From that dialogue grew the seeds for The *PA Mathematics Education Coalition.* This joint effort between PCTM, PAMTE, and PCLM has as its central mission informing state and local policymakers and the public on all aspects of high-quality mathematics education and advocating for policies that will improve mathematics education at every level.

When we think of the membership benefits of a professional organization we often call to mind tangible items like journals, publications, and conferences. Equally important, though, is the intangible work that a professional organization can do on behalf of championing the profession. The Math Ed Coalition has the potential to be a strong vehicle for growing our professional capital. It is the leadership's belief that the formation of a coalition is a first-step in increasing its influence at the state level in the arena of educational policy. While the work is merely in its infancy, the Coalition endeavors to become a known, trusted, and authoritative voice on mathematics education in Pennsylvania. Like any worthwhile effort, this can only be accomplished by strengthening connections and human relationships at the local, grassroots level. For the Coalition to have success, we will need the active involvement of PCTM, PAMTE, and PCLM members across the commonwealth reaching out to their legislators, school boards, administrators, and communities.

To this point the Coalition's work has been two-fold. First, it has largely been organizational. Representatives on the Coalition from all three organizations are working to develop a robust mission statement and action plan, researching efforts by similar groups in states across the U.S., and making initial contacts to state-level officials. Second and critically, the Coalition has worked hard to become engaged in the discussion of thoughtful state-level implementation of the *Every Student Succeeds Act (ESSA)*, a reauthorization of the Elementary and Secondary Education Act signed by President Obama on December 10, 2015. To that end, the Coalition has arranged for a special presentation by PA Department of Education on ESSA for mathematics education community. This meeting will be held on February 9, 2017 (6:30pm) at Messiah College, Mechanicsburg. Please see flyer on page 27 for details. The meeting will allow our members to become informed and to offer input into the implementation of the law which will impact our work for the foreseeable future. The PCTM membership's support of and attendance at this meeting is vital to the initial impact of the PA Mathematics Education Coalition. Together we are stronger. The New Year is a time for optimism and action! Please join the PA Mathematics Education Coalition in advocating for an exciting mathematics education future. We look forward to seeing you on February 9th!

Quote: *The first requisite of a good citizen in this republic of ours is that he shall be able and willing to pull his own weight. (Theodore Roosevelt)*

Katherine Remillard is a faculty member at Saint Francis University. She is President of PAMTE.

FRINGE THOUGHT: Many ideas grow better when transplanted into another mind than the one where they sprang up. –Oliver Wendell Holmes

College Corner: Cardano's Formula—An Italian Mathematical Melodrama in Three Acts

Mary Ann Matras

Prologue: Modern Pre-calculus students sometimes struggle with the sections in the course where they apply the Rational Root Theorem and Descartes Rule of Signs to find the roots of higher order polynomials. After all, they argue those theorems are not really necessary in the day of the calculator and computer. The texts and teachers persist, however, in looking at the older methods because those methods give the students a deeper understanding of how polynomials actually work. Perhaps it would be interesting to take a historical look at the search for formulas for finding the roots of polynomials in the days of hand calculations.

The simplest polynomial equation, beyond the linear equation which have been worked with since the time of the Egyptians, are quadratic equations. The Babylonians, Egyptians, and Euclid all used geometric methods to solve quadratic equations. The Chinese book from about 200 CE *The Nine Chapters on the Mathematical Art* had rules for working with some quadratics. The Indian mathematician Brahmagupta about 628 CE published a form of the quadratic formula. The quadratic formula that is part of our curriculum today was first published by Simon Stevin in 1594. After the quadratic, mathematicians then began looking for the general solution of the cubic.

The search for the general solution of the cubic began in earnest in the fifteenth and sixteenth century in Italy among a group of Italian mathematicians called the Abacists. At that time, Italian universities were not organized as modern universities. Professors did not have permanent jobs and often needed to have a mentor to help pay their salaries and ensure their continued employment. Mentors were royal or rich or both. But these mentors wanted something for their money and support so they often arranged contests where the mathematics professors solved problems in public for large audiences. Much wagering accompanied these contests. The professors, when they discovered something new, never shared their methods but saved them to use in future contests.

The melodrama begins in 1494 with the Italian abacist Luca Pacioli in his book *Summa se Arithmetica*. Pacioli had carefully written all about linear and quadratic equations, but he was convinced that there was no general solution for cubic equations and he challenged the other abacists to find such a solution.

Act 1: Scipione del Ferro (1465-1526), of the University of Bologna took up Pacioli's challenge. He was able to find a general solution only to a "depressed cubic"—one of the form $ax^3 + cx + d = 0$. Before his death, del Ferro kept this solution to himself and only revealed it to his pupil Antonio Maria Fiore and to his successor at Bologna Annibale della Nave. Neither man let it be known that he knew del Ferro's work but word got around of a possible solution.

Also working on the cubic was Niccolo Tartaglia (1499-1557). He had discovered the solution to cubics of the form $ax^3 + bx^2 + d = 0$. Fiore hearing the rumors in 1535 challenged Tartaglia to a public contest. Fiore thought he would win with del Ferro's formula. Each sent the other 30 problems. Fiore sent only ones that required del Ferro's work. Tartaglia, the better mathematician, discovered the solution of the thirty but Fiore was unable to solve many of Tartaglia's problems which covered a wide range of mathematics. Tartaglia won.

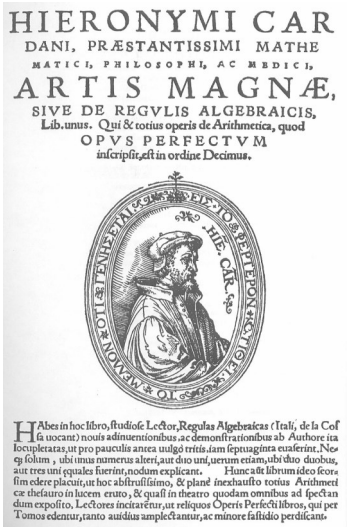
Nicolo Tartaglia
1500 - 1557

Hieronimo Cardano
1501 - 1576

FRINGE THOUGHT: Because things go wrong is no reason you must go with them.

Act 2: Word of the contest and the winner spread throughout Italy. Gerolamo Cardano (1501-1576) was a physician, astrologer, and gambler with a passion for mathematics. He was lecturing on mathematics and writing a book. Cardano wrote to Tartaglia many times begging to be told the solution. Tartaglia refused, but when Cardano moved to Milan and they met, Tartaglia finally relented and told the solution to Cardano, making Cardano promise never to publish the formula. Tartaglia was planning to write a book himself.

Cardano working with his pupil Ludovico Ferrari (1522-1565) finally discovered the solution to the general cubic equation: $ax^3 + bx^2 + cx + d = 0$. But Cardano's solution depended on Tartaglia's work with the depressed cubic and he could not publish it because of his promise to Tartaglia.


HIERONYMI CAR
DANI, PRÆSTANTISSIMI MATHE
MATICI, PHILOSOPHI, AC MEDICI,
ARTIS MAGNÆ,
SIVE DE REGVLIS ALGEBRAICIS,
Lib. unus. Qui & totius operis de Arithmetica, quod
OPVS PERFECTVM
inscripsit, est in ordine Decimus.

HAbes in hoc libro, studiose Lector, Regulas Algebraicas (Itali, de la Cosa uocant) nouis adinuentionibus, ac demonstrationibus ab Authore ita locupletatas, ut pro pauculis antea uulgo tritis, iam septuaginta euaserint. Neq; solum, ubi unus numerus alteri, aut duo uni, uerum etiam, ubi duo duobus, aut tres uni æquales fuerint, nodum explicant. Hunc aut librum ideo seorsim edere placuit, ut hoc absolutissimo, & planè inexhausto totius Arithmetice thesauro in lucem eruto, & quasi in theatro quodam omnibus ad spectandum exposito, Lectores incitarentur, ut reliquos Operis Perfecti libros, qui per Tomos edentur, tanto auidius amplectantur, ac minore fastidio perdiscant.

Remembering the rumors that del Ferro had solved the depressed cubic, Cardano and Ferrari traveled to Bologna where Fiore allowed them to read del Ferro's work. Considering that they got the information from del Ferro's notebooks made it acceptable to forget their promise to Tartaglia, and Cardano published his book *Artis Magnae* in 1545. In his book, Cardano published the solution to the depressed cubic with credit to del Ferro, his solution to the general cubic, and Ferrari's solution to the quartic.

Act 3: Feeling betrayed, Tartaglia was furious. Cardano had acknowledged him for his work but Tartaglia had intended to win contests and write a book based on his work. He was a poor professor who needed to supplement his pay. The following year Tartaglia did publish his work in *New Problems and Inventions.* Tartaglia wrote to Cardano who ignored him. Tartaglia was still short of money and in 1548 he challenged Ferrari to a public contest. Tartaglia clearly expected to win; he was a very accomplished mathematician. But Ferrari knew the general solutions of both the cubic and the quartic. At the end of the first day of the contest, Tartaglia knew he was beaten and he left town without completing the contest.

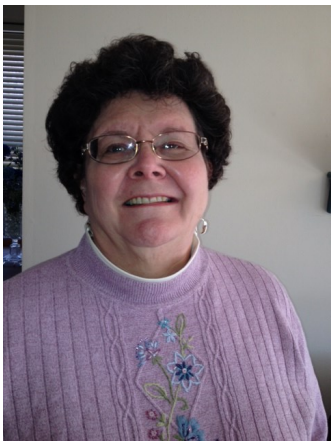
Epilogue: At the end of this mathematical melodrama, the solutions for the general cubic and quartic had been discovered and published. Cardano was a famous mathematician; Tartaglia was a beaten professor without a job. The cubic formula is usually called Cardano's formula although some modern writers have named it the Cardano-Tartaglia Formula so that while Tartaglia is not left out, he is still second best.

The search for solutions for general equations for polynomials of degrees greater than four continued after the Italian Abacists. Gauss was convinced that there is no solution for polynomials greater than four but he was unable to prove his assentation. It was proved in the 1820's by Niels Henrik Abel (1802-1824).

Read More About It:

Any good book on the history of mathematics contains the story of the cubic. I have used *A History of Mathematics* by Victor J. Katz (2004, Pearson Education), *The History of Mathematics: An Introduction* by David Burton (2007, McGraw Hill), and numerous internet sources. A search on the Internet of the names involved will produce many good (though some are rather fanciful) sources.

Mary Ann Matras is a Professor of Mathematics and the Chair of the Mathematics Department at East Stroudsburg University.



FRINGE THOUGHT: Happiness is a by-product of trying to make someone else happy.

Assessing Assessment as a Factor in Students' Transition to College-Level Mathematics

Ms. Jodie Styers and Dr. Courtney Nagle

"But I got an A in AP Calculus in high school!" This phrase is uttered in numerous college mathematics classrooms across the country countless times a semester. In an era where so much focus is being placed on mathematics, students are encountering more content than ever. Every high school graduate must complete the Keystone Exam in Algebra I (Pennsylvania Department of Education website) and the number of high schools offering Advanced Placement Calculus for students has increased dramatically (Mathematical Association of America, 2015). As high school graduates begin to transition to college, most encounter a mathematics placement exam. Students' scores on this exam facilitate placement into the correct entry-level college math class. For some students, this first semester class will be College Algebra. For others, it will be Calculus I. Arguably, the average first-semester college student has already encountered some of the curriculum in his/her first-semester math class. So why do so many first-semester college students struggle and under-perform in their math class? There must be something other than the curriculum that impacts students' transition between academic levels.

In 2015, the authors, Courtney Nagle (Assistant Professor of Mathematics Education) and Jodie Styers (Lecturer in Mathematics Education), submitted a grant proposal titled *Collaborations Between Academic Levels to Promote Successful Student Transitions from Secondary to Post-Secondary Mathematics* (Project #1544406) to the National Science Foundation. The project was fully funded ($49,956) and allowed the research team to investigate which factors might influence students' ability to be successful as they transition from high school-level to college-level mathematics. As part of the research project, a professional development cohort of mathematics educators was assembled. The cohort consists of six high school (grades 7-12) educators from four school districts paired with six college instructors from five institutions. The teachers selected for the cohort teach students in transitionary classes (i.e. first class in high school, last class in high school, first class in college). Participants attend monthly workshops to discuss items such as the assumptions, common practices, and constraints present at each academic level.

After one such workshop, cohort participants were challenged to compare and contrast assessment by responding to the question, "What is the role of assessment in your classroom?" Each cohort participant submitted a written response, generally 2-3 paragraphs in length, outlining the role that assessment plays in his/her classroom. In preparation for the second workshop, which centered on the idea of assessment, we used wordle.com to create a visual representation of each academic level's perception of assessment. In order to do this, we combined the responses from each academic level into one document, editing to remove non-imperative words and formatting issues (e.g., capitalization and plurals). The resulting Wordles provided a snapshot of the key terms that were used in the teachers' descriptions of assessment, highlighting the frequency of each term based on its size.

At the second workshop, we began our discussion of assessment by grouping the teachers by academic level and then gave each group the Wordle for the other academic level. We explained what the Wordles represented and asked each group to consider the following questions:

- Are there any words/phrases on the Wordle that you aren't familiar with?
- What words are you surprised to see? What words are you surprised you did not see?
- What do you think will be MORE prominent on your academic level Wordle?
- What do you think will be LESS prominent on your academic level Wordle?
- Develop three questions that you would like to ask the other academic level about their assessments.

This activity spurred a discussion on assessment and allowed an opportunity for participants to reflect on what struck them from the other academic level even before they saw their own level's Wordle. The college instructors had questions about some of the terminology used by the high school teachers, for example CDT and exit ticket. The high school teachers' Wordle (shown in Figure 1) provided evidence of multiple forms of assessment such as exit ticket, homework, formative, summative, and mini-meeting.

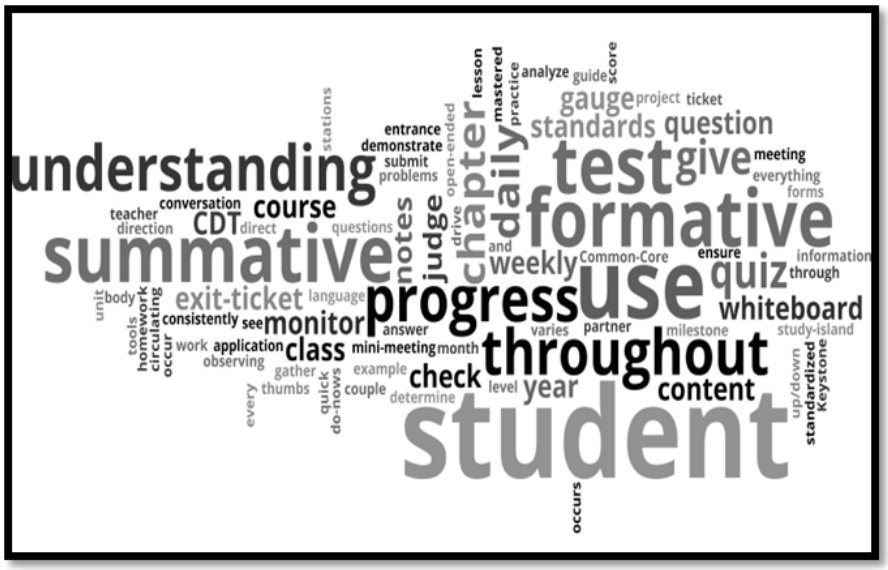


Figure 1. High school teachers' assessment Wordle.

The Wordle constructed from the college instructors' contributions (shown in Figure 2) focused much more heavily on exams with small mentions of homework and presentations. This suggests that the two groups rely on different forms of primary assessment to determine if students understand the material. The most noteworthy similarity between the two Wordles was the prominence of the word 'student'. Despite the different forms of assessment implemented at each academic level, the shared focus was the students. Every instructor in the cohort was adamant that his/her assessments were selected to monitor student progress.

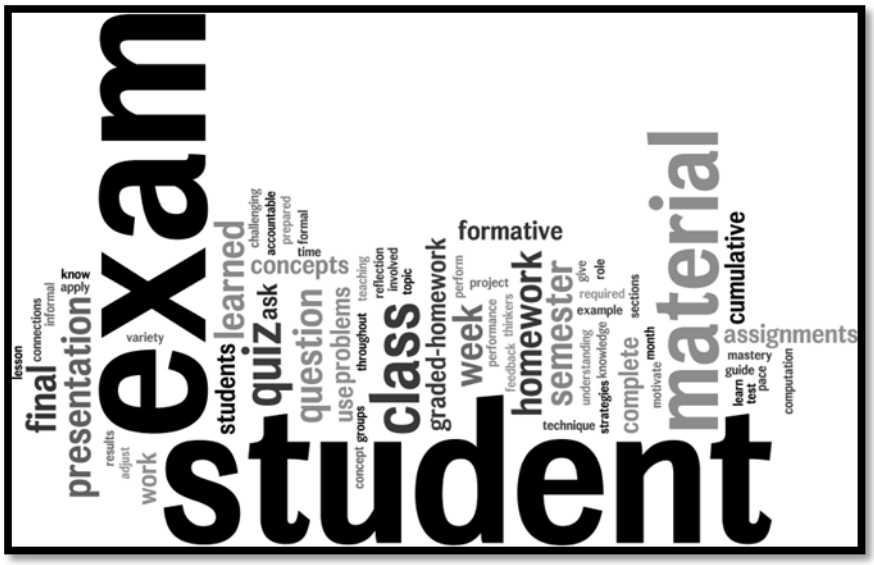


Figure 2. College instructors' assessment Worlde.

After discussing the role of assessment through analyzing the Wordles, the teachers then shared actual class assessments that they used in their math classes. Each academic level examined the assessments used at the other academic level and again generated thoughts and questions for their peers at the other academic level. A rich discussion ensued, covering ideas such as why there were so many multiple-choice questions on the high school tests (i.e., preparing students for standardized tests), whether students were permitted additional time to take exams, if and how review sessions were handled, etc. The sample exams provided evidence of a greater variety in questions (multiple-choice, open-ended, etc.) and a greater emphasis on procedure at the high school level. In contrast, the college-level exams demonstrated less variety in question strategy but a greater occurrence of conceptually-based questions.

Another point of discussion was the use of test retakes. The high school teachers reported that permitting a student to retake an exam to improve his/her score is a common practice. The cohort of college instructors was mostly unaware that the practice existed or was so prevalent at the high school level. Furthermore, they indicated they do not allow students to retake exams under any circumstances.

While assessment may only be a small part of the transition puzzle, it does provide valuable insight into the differences that exist between the high school and college mathematics classroom. Differences beyond assessment exist and still need to be explored. Once the differences are identified, perhaps we, as a community of educators, can better understand the transition through which students must navigate in order to experience success at both academic levels.

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Pennsylvania Department of Education. *Graduation requirements: Frequently asked questions.* Retrieved from http://static.pdesas.org/Content/Documents/Pennsylvania_Graduation_Requirements_Frequently_AskAs_Questions.pdf .

www.wordle.net

Jodie Styers is a lecturer in Mathematics Education and serves as undergraduate coordinator of the secondary mathematics education program at Penn State Behrend.

Courtney Nagle, Ph.D., is an assistant professor of Mathematics Education and program chair of the secondary mathematics education program at Penn State Erie, The Behrend College.

FRINGE THOUGHT: The noble secret of laughing at oneself is the greatest humor of all.

If you are interesting in becoming a member of or renewing your membership to PCTM, please go to the website or send an email to pctmmembership@gmail.com.

Navigating Your Way Around the 100s Chart

Jo Kinsey

One tool I have found to be valuable in the classroom is the 100s chart. See Figure 1. At the beginning of the year, each student in my classroom is given a 100s chart that has been laminated. This chart is kept in each child's desk in a special math folder which also contains a laminated place value chart, a ten frame, and a double ten frame. As the year progresses and our math knowledge increases, more tools are added to this folder. Lamination allows pupils to write on these tools with a dry erase marker.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 1. Hundreds chart.

The first thing we do is look at the 100s chart. Children are quick to observe patterns. "All the numbers end in 0 in the last column," is an oft-heard remark. "When I go across a row, the number in the last place, goes up," is also an observation. This is the time to introduce the terms "column" and "row."

In the beginning of the year, I formatively assess children's number knowledge by asking them to locate specific numbers. Or I ask them to silently point to a number that is greater or less than a number. I also use the 100s chart when skip counting. Children have a visual aid when they color in the numbers that show skip counting by 5s or 10s. Using the chart like a huge number line, they learn to wrap around the chart as they skip count and color in by 3s, 7s, or whatever number they choose. Children love puzzles. A great and fun activity I do with my students is Where Am I? using the 100s chart. I give my students a starting number such as 53. On the whiteboard I follow the number with a series of arrows, which point up, down, right, or left. The students trace the path of the arrows on the chart and find the ending number in the series. See Figure 2 for an example.

Students quickly become adept at navigating to numbers using the arrows. They intuitively realize the down and right arrows are addition; the up and left arrows are subtraction. Some students come to understand that in a lengthy series of arrows, an up and down arrow will cancel each other out and the left and right arrows will do the same. They also enjoy creating a series of directions, which others can solve.

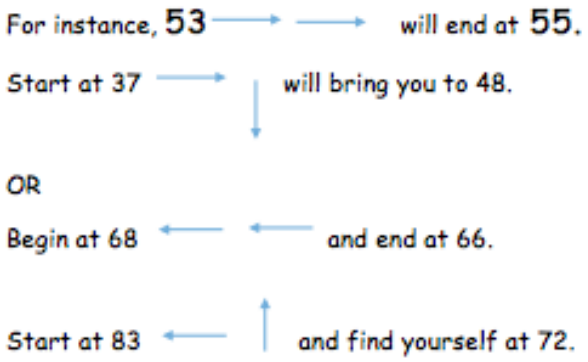


Figure 2. Examples for the activity Where Am I?

The 100s chart can also be used as a preliminary activity for adding and subtracting with place value. As student math knowledge increases and their understanding of tens and ones does also, you can add and subtract. For example, 23 + 56 can be solved using the 100s chart. Starting at 23, children add on the 6 ones by going "across" six spaces. They arrive at 29. Then they add on the 5 tens of 56 by "going down" 5 spaces in the column. They end up on 79.

Another number game activity I use is "Mystery Number," which gives children clues to find a specific number using the 100s chart. Children can use their dry erase markers to mark parameters. Here again, students love solving puzzles. An example of clues used to locate a number in the "Mystery Number" activity is:

- The Mystery Number is greater than 20.
- It is less than 30.
- It is an even number.
- When I add the digits together, the sum is 8.

These are a few activities to make math fun for your students using a 100s chart. The tasks can be used as a lesson, as part of a lesson, or as an extended activity. Enjoy!

Jo Kinsey is a First Grade teacher at West Bradford Elementary School in the Downingtown Area School District. She serves as Awards Chair for PCTM.

FRINGE THOUGHT: Many can rise to an occasion, but few know when to sit down.

A Historical Statistical Journey Through Data, Chance, and Uncertainty

Kevin Robinson

Winter 2017 – Happy New Year! Since the November election, recent news headlines have included *"What will pollsters do after 2016?"* and *"How did polls miss the presidential election result so badly?"* reminds us that data, chance, and uncertainty are as important as ever. The headlines also bring to mind that the grand statistical endeavor of inference from a sample to a population is hard. In this column, we will continue our historical statistical journey by meeting an individual who was central in the development of the modern day field of Statistics.

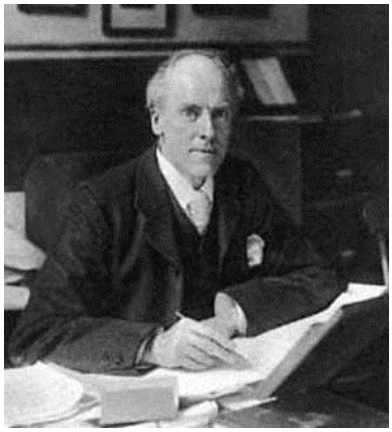


Figure 1. Karl Pearson (1857 – 1936).

The Englishman Karl Pearson (see Figure 1) was Francis Galton's statistical heir and a major player in the early development of Statistics as a serious scientific discipline in its own right. He founded the Department of Applied Statistics (now the Department of Statistical Science) at University College London in 1911; it was the first university Statistics department in the world. He had become interested in the idea that sound mathematics could be applied to natural phenomena not only under the category of causation, but also under the broader category of correlation. In 1893, he contributed his first statistical paper to the Royal Society, of which he was elected a Fellow in 1896. Pearson is most widely known to posterity as the inspirer and creator of a large body of statistical theory including frequency curves, correlation, and goodness of fit—most of which has appeared in the journal *Biometrika* (see Figure 2). *Biometrika* was established in 1901 by Karl Pearson and others to promote the study of Biometrics after the Royal Society had "resolved that mathematics and biology should not be mixed," as Pearson himself phrased it (Yule & Filon, 1936, p. 77).

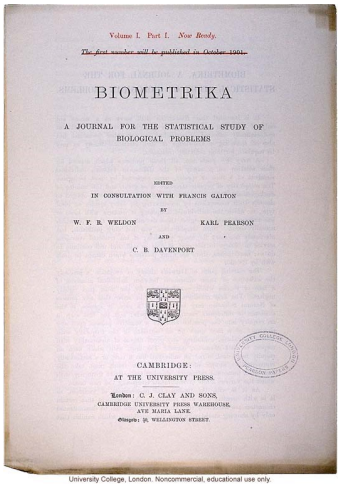
One of Pearson's most lasting contributions would be his development of Pearson's chi-squared test used to assess two types of comparison: tests of goodness of fit and tests of independence. See Figure 3. The chi-square test is commonly used to discover if there is an association between two categorical variables. For example, consider investigating with your students if the variable: coke/pepsi preference is associated with the variable: corrective eyewear yes/no.

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Figure 3. Pearson's Chi-Squared Test Statistic. Note: O = Observed E= Expected.

In the next column, we will learn who the original Mr. T was and why his work was so crucial in the development of the commonly used t-test for means. Until then, wishing you a safe and happy new year 2017. Reminder: a number of resources, including a timeline of statistics, are the following website: http://sites.millersville.edu/krobinson/STATHIST/.

Statistically Yours ~ KSR



Volume I. Part I. *Now Ready.*

~~The first number will be published in October 1901.~~

BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

EDITED
IN CONSULTATION WITH FRANCIS GALTON
BY
W. F. R. WELDON KARL PEARSON
AND
C. B. DAVENPORT

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Figure 2. Biometrika 1st Issue 1901 – A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS.

Yule, G. U. & Filon, L. N. (1936). Karl Pearson 1857-1936. *Obituary Notices of Fellows of the Royal Society*, 2(5), 77.

Kevin S. Robinson, PhD, is an Associate Professor in the Department of Mathematics at Millersville University of PA. He is a statistical educator with interest in K-16 statistical curriculum and the application of industrial statistics.

FRINGE THOUGHT: Words are like a prescription dosage: More is not necessarily better.

Proactive and Professional: Learning From Each Other

Annette Cook

Where do teachers get new teaching ideas? In-service? Pinterest? Graduate classes? Conferences? Math magazines? While teaching is thought to be an individual endeavor—one teacher with lots of students, collaboration with other teachers is vital to my continued growth as a classroom instructor. I have learned so much about middle schoolers, classroom management, and effective communication with parents and administrators by working with teachers that are inside and outside of my subject area. But, more importantly, I get great instructional ideas from other math teachers. Educators want to share what works for them in the classroom. Often, these ideas are easy to use, and they resonate with the students. This article is about two quick teaching tips that were shared with me from colleagues. Sometimes, a small idea can make a big difference in approaching the teaching of a lesson and reinforcing the learning of the students.

An Idea from an Elementary Teacher

Mrs. Fox is an elementary math teacher in my district. Our elementary buildings house students from kindergarten to sixth grade. Mrs. Fox teaches pre-algebra to sixth graders. These are very bright students, and yet their age and brain development may hinder their understanding of abstract ideas introduced in a pre-algebra course. This is also an issue in my eighth grade pre-algebra classroom.

Students can typically identify a slope as being positive or negative when given the graph. It is those pesky zero and undefined slopes that cause confusion. Mrs. Fox calls upon the math superhero *Slope Man* to the rescue. See Figure 1. Notice the eyes and eyebrows. These show, from left to right, positive slopes go up and negative slopes go down. The mouth is a horizontal line with zeros on each end. Slope Man's mouth indicates horizontal lines have a slope of zero. The nose has a "U" at the bottom. The nose is a vertical line and, therefore, is a reminder that all vertical lines have an undefined slope. Creative!

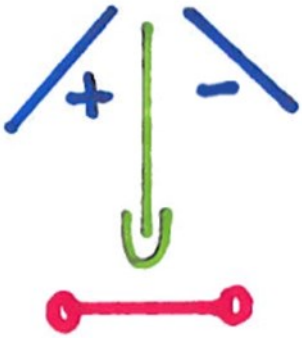


Figure 1. Slope Man.

An Idea from a Middle School Teacher

Mrs. Rice is a retired middle school teacher. She taught in one of the middle schools in my district; I taught in the other. When teaching angle relationships, we typically teach supplementary and complementary angles in the same unit. Early in my career, I would try to use alphabetical order to help the students remember the difference between these terms. "C" comes before "S" in the alphabet and 90 is less than 180. Therefore, complementary angles are two angles that sum to 90˚ while supplementary angles are two angles that sum to 180˚. Ninety is less than 180 and "C" comes before "S." This makes sense but, then again, I do not get these two terms confused.

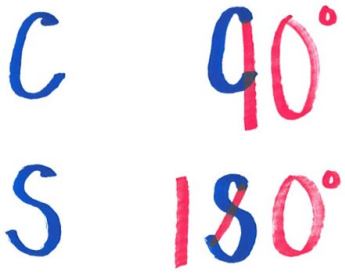
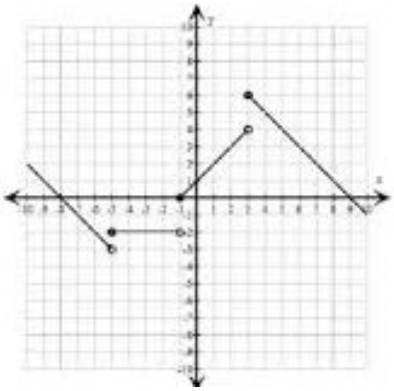


Figure 2. A visual used to help students remember the sum of complementary and supplementary angles.

Mrs. Rice showed me this visual. See Figure 2. The word "complementary" begins with the letter "C." If you add one line to a "C," you get a "9." This helps students remember that complementary angles sum to 90˚. Likewise, "supplementary" begins with the letter "S." If you add one line to the "S" you get an "8." This helps students remember that supplementary angles sum to 180˚. Ingenious!

Do you have a nifty classroom instructional idea or a clever activity to share? I invite you to write an article and submit it to the editors of the PCTM magazine at pctm.editor@gmail.com.



Annette Cook is a 7th and 8th grade mathematics teacher at Landisville Middle School in the Hempfield School District. She currently serves as a PCTM Membership Chair.

FRINGE THOUGHT: It's not the difference between people that's the difficulty. It's the indifference.

Number Sense with 2017

Tim Seiber

Happy New Year 2017$_{tws}$

Every school year there are moments that provide opportunities for teaching or reviewing content in a different manner. This is one such opportunity.

First, a little background. During the months leading to the year 2000, there was a lot of concern about what might happen with programs and computer systems designed to only use the last two digits of a calendar year. Would using "00" cause computers to treat the year as 1900? Would the millions of computer calculations done each day by financial institutions suddenly be based on the year 1900, train and plane schedules based on the year 1900? Every news broadcast talked about the possible outcome as the countdown to 2000 occurred. Thankfully most computer software was updated prior to 2000. If you are a little young to remember this you may be surprised that even now we see the results, no more 2 digit year entries. Years are now inputted using all 4 digits.

This was the first time I asked my students to tell me about the number used for a new calendar year. Every year since 2000, I have begun the calendar year asking students to tell me about the number. Sometimes it is in the form of a few questions to start class, other years it is more formal. This year is a more formal exploration of the number.

Some numbers are rich in factors and patterns. Other numbers have few factors but offer other opportunities, 2017 is one such number. The activity sheet is based on content from the middle school common core curriculum and the activity is designed to provide the opportunity for students to think. You will notice more prime numbers are listed for testing 2017 than are needed. Did the students read the instructions on how to test for prime? This is an opportunity to talk about the meaning of "half way." This year I had a student ask why we did not test all of the numbers until we got to half of 2017. The two of us had a nice discussion on the meaning of "half way." After using 36 and its factors, he quickly understood the "half way" was not half of 2017 but rather half of the factor list.

Also, calculating the square root is done later in the activity. Have you noticed sometimes on open-ended standardized test question part b or c is helpful when answering part a?

The activity provides the middle school student with an opportunity to learn or review the cube and cube root keys on their calculator. Using their phone calculator apps may be ok for some activities. However if you wish your students to know their keys for standardized testing, they need an opportunity to practice using their calculator. This provides that opportunity.

You are encouraged to change and adapt this activity to meet the needs of your students. Enjoy!

FRINGE THOUGHT: Confidence is contagious. So is lack of confidence.

Tim Seiber is a teacher at Eagle Valley Middle School in Mechanicsburg, PA. He serves as a Board Member for PCTM.

Happy New Year 2017$_{tws}$

A little number sense about 2017. What do we know about this number?

Is it even or odd?

Is it divisible with no remainder for the "classics"?

2? 3? 5? 10?

What is the remainder for any of these numbers that do not have a remainder of zero? Not the decimal, the remainder please.

÷ 2 r = ÷ 3 r = ÷ 5 r = ÷ 10 r =

2017 may be prime! To test for prime you must check every prime number less than the square root of 2017.

Check as many of these primes as needed. Circle any number that is a factor of 2017.

2,	3,	5,	7,	11,	13,
17,	19,	23,	29,	31,	37,
41,	43,	47,	53,	59,	61,
67,	71,	73,	79,	83,	89,
97,	101,	103,	107,	109,	113

FRINGE THOUGHT: You never learn anything talking. You learn only when you ask questions.

What was the largest number you tested?

What is the square root of 2017?

What are the two perfect squares immediately above and below 2017?

Could there be a perfect cube root for 2017?

What is the cube root of 2017?

What are the 2 perfect cubic numbers immediately above and below 2017?

Let's go to the opposite operation, squares and cubes.

Sometimes squaring or cubing a number creates a pattern,

$$11^2 = 121, \quad 11^3 = 1331.$$

What is the square of 2017? Do you see a pattern?

What is the cube of 2017? Do you see a pattern?

Your thoughts about the number 2017?

FRINGE THOUGHT: You'll be happier if you give people a bit of your heart rather than a piece of your mind.

Counting in Chinese

Lara Dick

My middle daughter, Irene, is 4 years old and is thus in the process of beginning to understand counting numbers as representative of actual quantities. Irene has an advent calendar that contains a piece of chocolate that she gets to eat each morning (oh, the joy!). This week, we hit our difficult numbers (11, 12, 13, 14, 15, etc.). While I know these are often hard numbers to understand because of the structure of our English language, I had forgotten that this would be a struggle for her each day. When we've said, for example, "Irene, find the eleven. It's a ten and a one," she can find lots of ones on her calendar, but doesn't yet conceptualize 11 as composed of a 10 and a 1, mostly because eleven doesn't sound like a ten and a one. Throughout the week, I've been reminded of how we worked through this with our oldest daughter, Claire…we taught her Chinese numeration.

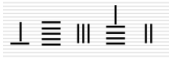
The current Chinese numeration system is based on Chinese rod numerals, which were developed over two thousand years ago (Merzbach & Boyer, 2010; Zaslavsky, 2001). In this base 10 system, there was a bamboo stick, or rod, representation for each of the digits 1 to 9 (see Figure 1) as well as the first nine multiples of 10 (see Figure 2). With only these eighteen symbols, all numbers were represented by alternating horizontal and vertical rods on a board with a spot for each place value. For example, the number 65,392 looked like  but it was placed on a board indicating the value for each of the digits. You may be thinking, that's pretty much exactly how we think about and write our Hindu-Arabic numbers, which is partially true. But what about how the Chinese numbers were said aloud?

Figure 1: Chinese rod numerals for digits 1 to 9.

Figure 2: Chinese rod numerals for the first nine multiples of 10.

In Ancient China today, the written and spoken form of their numeration system indicates the number of units in each base, together with that base. For example, the number 87 is actually written and read as 8 tens and 7 ones. So for Irene, instead of saying eleven, if we were in China, we would literally say one ten and one when we say the number aloud. When the Chinese moved away from the counting board and rods, they replaced their system with numerals that indicated the place value (see Figure 3).

0	zero	零	10	ten	十
1	one	一	100	one hundred	百
2	two	二	1,000	one thousand	千
3	three	三	10,000	ten thousand	萬
4	four	四	100,000	one hundred thousand	十萬
5	five	五	1,000,000	one million	百萬
6	six	六			
7	seven	七			
8	eight	八			
9	nine	九			

Figure 3: The current Chinese numeration system (Yu, 2003, p. 244).

FRINGE THOUGHT: There's nothing worse than being a doer with nothing to do.

Yu (2003) explains this system with an example comparing our Hindu-Arabic system to that of the Chinese. He states,

> Consider the number 564. The "5" has a value of 5 x (10^2), or 500; "6" has a value of 6 x (10^1), or 60; and "4" has a value of 4 x (10^0), or 4. In the Hindu-Arabic system, the base (10) is implied. Written in Chinese characters 五百六十四 , 564 is (5; 100; 6; 10; and 4), which literally means five hundred(s), six ten(s), and four. (p. 244)

Yu goes on to discuss how those who use Hindu-Arabic numbers, can rewrite our numbers in Chinese form by writing in their expanded form.

With all of this in mind, this week, I've started working with Irene. I've told her that our numbers have nicknames and that eleven is also called ten one and twelve is also called ten two. Today is December 15th, and fifteen is also called ten five, which we can write as 15 or 10 + 5. It's a process that will continue for Irene. But for her older sister, Claire, who is in second grade, it's a reminder of how to use expanded form for operations, and once again how much more natural this is for students who speak Chinese. When working with Claire doing problems like the ones seen in Figure 4, it has been helpful to remind her of Chinese numbers as she is saying and solving problems:

332	*3 hundreds, 3 tens and 2 ones*
+ 457	+ *4 hundreds, 5 tens and 9 ones*
	7 hundreds, 8 tens and 11 ones
	700 + 80 + 11
	791

Figure 4: Example of how to "say" Chinese numbers while operating with expanded form.

Our girls enjoy learning about how a different culture writes, says, and thinks about numbers as I am sure many of your students will as well. I encourage you to read more about the Chinese numeration system in the two *Teaching Children Mathematics,* a NCTM publication for practicing teachers, articles referenced below. It's yet another way to make math fun, but also easier to understand.

References

Merzbach, U. C. and Boyer, C. B. (2010) A History of Mathematics. 3rd. Edition. John Wiley & Sons, Hoboken, NJ.

Uy, F. L. (2003). The Chinese numeration system and place value. *Teaching Children Mathematics*, *9*(5), 243–247.

Zaslavsky, C. (2001). Developing number sense: What can other cultures tell us? *Teaching Children Mathematics*, *7*(6), 312–319.

Lara Dick is an Assistant Professor in the Math Department at Bucknell University.

The ***Pennsylvania Mathematics Education Coalition*** is pleased to host

Ms. Beth Olanoff

Special Assistant to the Secretary

PA Department of Education

For an overview and discussion of

PA's state plan for implementation of the *Every Student Succeeds Act (ESSA)*

Thursday, February 9, 2017

6:30 p.m.

Messiah College, Kline Hall Room 120

1 College Avenue Mechanicsburg, PA 17055

WHO: This meeting is open to all professionals with an interest in high-quality mathematics education: K-12 teachers, mathematics teacher educators, special educators, curriculum specialists, and administrators. All are welcome!
MAP: Campus map http://www.messiah.edu/map/
RSVP: An RSVP is encouraged to help for planning purposes but *not required*. Those not able to attend are invited to submit their questions and comments for PDE. To RSVP and/or provide questions/comments go to: https://goo.gl/forms/oWvNtVUmio5UgDO72
INCLEMENT WEATHER: A cancellation of the meeting due to inclement weather will be posted not later than 3 p.m. on the PAMTE website. Visit http://www.pamte.org/ and view *Schedule of Events* (lower left corner of homepage).

This program is brought to you by the **Pennsylvania Mathematics Education Coalition** partners**:**

with generous support from **Messiah College**:

FRINGE THOUGHT: It is better to be hated for who you are than to be loved for who you are not.

Information about Upcoming Math Contests

	Name	Grades	Type	When	Where
1	MATHCOUNTS	6-8	individual and team	several times per year	School, Regional State, National
2	Challenge 24	3-8	individual	May	Check with Local IU for Regional Competition
3	Gold Exam at Bucknell University	9-12	individual and team	Registration mid October, March 2017 competition	Bucknell University, Lewisburg
4	IUP High School Mathematics Competition	9-12	individual	4/20/2017	Indiana University of Pennsylvania, Indiana
5	Lehigh University High School Math Contest	8-12	individual and team	3/4/2017	Lehigh University, Bethlehem
6	Marywood High School Mathematics Contest	9-12	individual	3/25/2017	Marywood University, Scranton
7	MCWP Algebra Contest	6-9	individual	4/8/2017	Mathematic Council of West Pennsylvania
8	MU High School Math Contest	10-11	individual and team	3/1/2017	Millersville University
9	PA Statistics Poster Competition	K-12	individual and team	2/27/2017	school, national level
10	Pennsylvania Math League	4-12	team	one or several times per year	your school, Stanford
11	Shippensburg University Annual Math/Computer Day	9-12	individual and team	4/7/2017	Shippensburg University

If you have additional contests or competitions that you would like us to include, please email Scott Breeden, PCTM Contest Chair at william.breeden@gmail.com.

FRINGE THOUGHT: Courage is not the absence of fear, but the ability to conquer that fear.

	Money Awards	Name of Sponsor	Contact	Phone	E-mail
1	up to $20,000 Scholarship	PA Society of Professional Engineers	Jennifer Summers	717-441-6051	jennifer@wannerassoc.com
2					
3		Department of Mathematics at Bucknell University	Polly Doyle	570-577-1343	math@bucknell.edu
4		Mathematics Department of Indiana University of Pennsylvania	Tim Flowers	724-357-4765	flowers@iup.edu
5		Lehigh University	Don Davis	610-758-3756	dmd1@lehigh.edu
6	$25-$2000 in money and scholarships	Marywood University	Marcie Gaughan	570-348-6265	mgaughan@marywood.edu
7	$100, $75, & $50 for first, second, and third place winners	Mathematics Council of Western PA	Robert Blamick		blamickr@aol.com
8		Department of Mathematics at Millersville University	Michael Wismer	717-871-7321	michael.wismer@millersville.edu
9	up to $200	Saint Francis University	Peter Skoner	814-472-3085	pskoner@francis.edu
10		Mathematics Leagues Inc.	Dan Flegler	201-568-6328	Dan@mathleague.com
11		Shippensburg University	Ben Galluzzo	717-477-1431	BJGalluzzo@ship.edu

FRINGE THOUGHT: One of the best things a person can have up his sleeve is a funny bone.

FRINGE THOUGHT: There are two kinds of fishermen—those who insist they fish for sport, and those who catch something.

Pennsylvania Statistics Poster Competition

A statistics poster is a display containing graphs that summarize data, provide different points of view, and answer some question (or questions) about the data.

Ordinarily, a $96 first prize, a $72 second prize, a $48 third prize, and a $24 fourth prize will be awarded in each of the four grade level categories.

Judges will look for the following:

- Overall impact of the display for eye-catching appeal and visual attractiveness, and for its ability to draw in the viewer to investigate the graph or graphs.
- Clarity in the demonstration of relationships and patterns, obvious conclusions, and the ability to stand alone, even without the documentation on the back of the poster.
- Appropriateness of the graphics for the data.
- Creativity, neatness, and originality.

- Open to all K-12 students in Pennsylvania
- Four grade level categories: K-3, 4-6, 7-9, and 10-12
- Cash prizes, certificates, and ribbons awarded for first-, second-, third-, and fourth-place in each category.
- Winning posters are also submitted to the national statistics poster competition.

Submission Deadline: Tuesday, February 28, 2017
Poster Judging: March 2017
Winners Announced: April 2017

Registration opens November 2016:

francis.edu/pa-statistics-poster-competition/

QUESTIONS? PLEASE CONTACT US:

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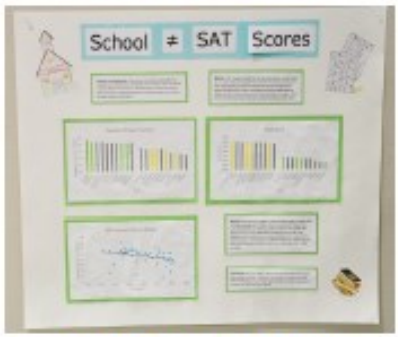
2016 PA Statistics Poster Competition Winners

Can You Really Improve Your Chances of Winning Rock-Paper-Scissors
1st Place, K-3 Grade Level

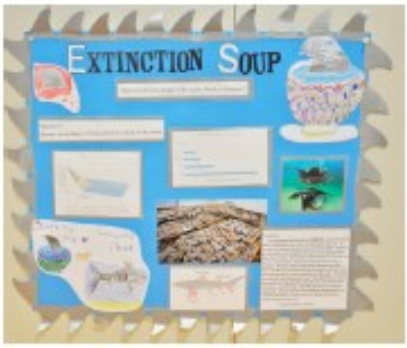
Cheesy and Sweet, Which Would You Like to Eat?
2nd Place, K-3 Grade Level

Do People Still Say Thank You
3rd Place, K-3 Grade Level

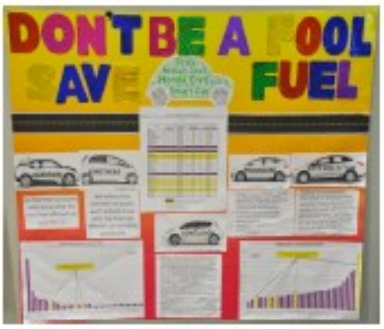


School Not = SAT Scores
1st Place, 4-6 Grade Level

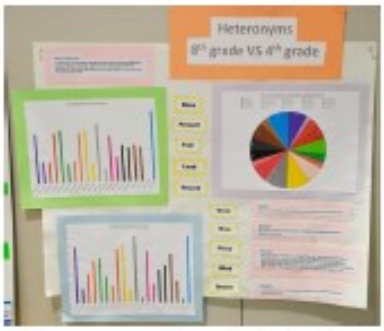
Fame or Fam?
2nd Place, 4-6 Grade Level



Extinction Soup
3rd Place, 4-6 Grade Level

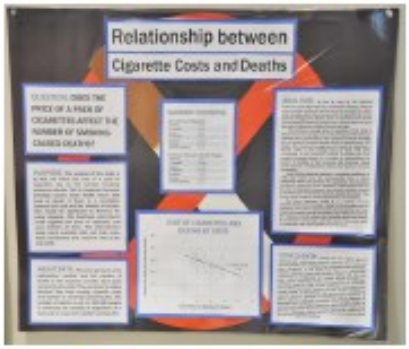


Don't Be A Fool, Save Fuel
1st Place, 7-9 Grade Level



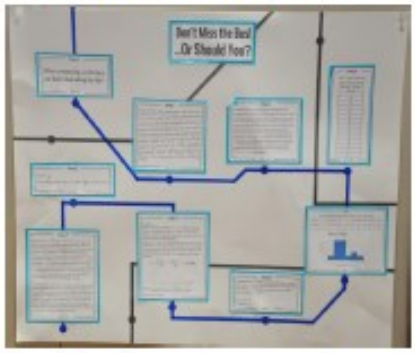
Heteronyms 8th vs 4th Grade
2nd Place, 7-9 Grade Level

E-books vs. Print Books
3rd Place, 7-9 Grade Level



Cigarette Costs and Deaths
1st Place, 10-12 Grade Level

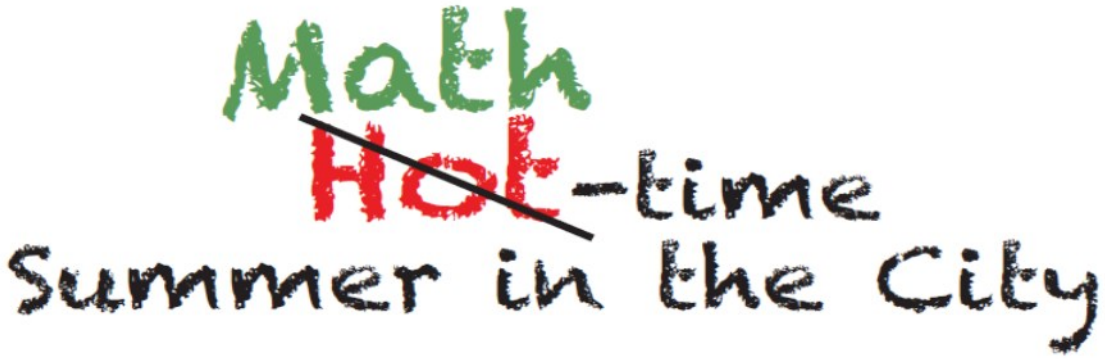
Gauging the Game
2nd Place, 10-12 Grade Level



Don't Miss the Bus...Or Should You?
3rd Place, 10-12 Grade Level

FRINGE THOUGHT: Rain is caused by high-pressure areas, cold fronts, warm, moist air, and weekends.

PCTM Annual Conference



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Teaching students to **pay attention to details** to solve problems

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FRINGE THOUGHT: One of the greatest labor-saving devices of today is tomorrow.

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FRINGE THOUGHT: More important than how we live is how we spend our day.

Submissions Solicited for PCTM Magazine

Since the 1990's, the Pennsylvania Council of Teachers of Mathematics (PCTM) has produced the PCTM Magazine for our members. Our mission is to promote mathematics education in Pennsylvania. In the magazine we accomplish this by publishing edited articles by leading authors and local news from around the state. PCTM is committed to improving mathematics instruction at all levels. We place an emphasis on classroom activities that are aligned to the Pennsylvania Core State Standards and the NCTM Principles and Standards for School Mathematics.

You are invited to submit articles for consideration for publication in the PCTM Magazine. This publication provides an excellent opportunity for you to share your ideas with the ever-growing number of colleagues dedicated to improving mathematics education in Pennsylvania. Any topic of interest to teachers of mathematics, especially K-12 classroom teachers in Pennsylvania, is suitable subject material. All readers are encouraged to contribute articles and opinions for any section of the magazine. Teachers are encouraged to submit articles for Voices From the Classroom, including inspirational stories, exemplary lessons, or management tools.

Original artwork on the cover is another way teachers may contribute. We publish the magazine three times each school year, in the winter, spring, and fall.

Deadline for submissions:

Spring, April 15
Fall, August 15
Winter, December 15

Author Guidelines:

Manuscript Format: Manuscripts should be double-spaced, with 1-inch margins on all sides, typed in 12-point font and follow the APA 6th Edition style guide. Manuscripts should be submitted in Microsoft Word. If you have a picture or graphic in the text, please include the original picture(s) in a separate file. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audience should be included in the e-mail.

Manuscript Submission: Manuscripts should be submitted electronically as an e-mail attachment to pctm.editor@gmail.com. Receipt of manuscripts will be acknowledged. After review by the editors, authors will be notified of a publication decision.

Dates of Upcoming Conferences

2017 T^3 International Conference, March 10-12, 2017, Chicago, IL

EPaDel (Eastern Pennsylvania Delaware section of the Mathematical Association of America), April 1, 2017, Kutztown University, Kutztown, PA

NCTM Annual Conference, April 5-8, 2017, San Antonio, TX

NPCTM Spring Meeting, April 27, 2017, Radisson Lackawanna Station in Scranton, PA

11th Annual PAMTE Conference, May 17-18, 2017, Shippensburg, PA

MAA MathFest, July 26-29, 2017, Chicago, IL

66th Annual PCTM Conference, August 2-3, 2017, Harrisburg, PA

NCTM Regional Conference, October 18-20, 2017, Orlando, FL

Preservice Teacher Day (for the Eastern part of PA), October 21, 2017, Millersville University

Preservice Teacher Day (for the Western part of PA), October 21, 2017, Indiana University of PA