An Official Publication of the

Pennsylvania Council of Teachers of Mathematics

FRINGE THOUGHT:

Cover credit:

Huffpost.com

Image created by walking in the snow.

Volume LVII

Number 2

Winter 2019

Table of Contents

3 President's Message
Kelly Brent
4 Welcome from the Editors
Debbie Gochenaur and Edel Reilly
6 Preservice Teacher Day at St. Joseph's University
Janine Firmender
7 Preservice Teacher Day at California University of Pennsylvania
Barbara Hess and Marcia Hoover
8 Social Justice in the Mathematics Classroom: A Critical Perspective
Ben Phillian, Yuliya Melnikova, and Janet Walker
15 Using Pattern Blocks to Understand Fractions and Fraction Operations
Karise Mace and Keri Stefkovich
26 Number Talks: How to Help Students Develop Number Sense
Valerie Long
33 Call for Statistics Poster Competiton
Peter Skoner
34 A Math History Book Review
Laura Dick
36 PCTM Honors Recent Retirees
Jane Wilburne
38 Upcoming Conference Dates

39 Affiliate Reports

42 PCTM Elections—Call for Nominations

43 Call for Manuscripts-Writing in the Math Classroom

PCTM Magazine is published three times each year by the Pennsylvania Council of Teachers of Mathematics, an affiliate of the
National Council of Teachers of Mathematics.

Articles and announcements for **PCTM Magazine**, an editor-reviewed publication, should be submitted electronically to the editors via pctm.editor@gmail.com

The **PCTM Magazine** editors are:
Debbie Gochenaur (Shippensburg) and Edel Reilly (IUP)

President’s Message

Kelly Brent

A new year has begun! For some of us, that means a fresh start to a new semester and for others, it means we are in the middle of our academic year. In either case, it is a nice time to reflect, revise if necessary, and move forward with enthusiasm.

I have kept Jim Rubillo’s message from his Keynote speech this summer in the forefront of my mind as I prepare lessons. Jim emphasized, “Our goal is to help students become confident problem solvers, not symbol manipulators.” This has proven to be a challenging task. It seems that at times I am pulling hard, practically forcing students to explore and ponder so that they can understand the “why” when, it seems that some of them just want to memorize the “what” in order to pass quizzes, tests or other assessments. This frustrated me until a student helped me to realize why they were hesitant to just “play with a problem” as I refer to it in class. The knowledge that eventually there would be a quiz or test on the material covered in class, makes some students anxious and wanting a formula or process to memorize so that they feel secure in what they are doing. They find comfort in memorizing the quadratic formula, hearing the words they cling to, “this will always work”.

So as I transition to the new year, I will continue to incorporate problem solving opportunities for my students, but now I am working on easing their anxiety around grading so that their “joy” in problem solving and discovery and inquiry, can be restored. This, of course goes hand in hand with a focus of NCTM’s *“Catalyzing Change in High School Mathematics”*

So my message to my fellow mathematics instructors, is simply this. Embrace what is working with your mathematics instruction as you move into the new year, but have the courage to change or revise what you know is not quite working. This is problem solving at its finest and a nice way to model it to your students. We try, try, and try again until we find solutions. Thomas Edison said it best, “If there is a better solution, find it.” As math teachers we will continue to try to make the process of finding the solutions fun, invigorating, challenging and filled with joy!

Happy New Year and I look forward to seeing you at the PCTM conference in August.

Kelly Brent
PCTM President
brentk@carlisleschools.org

FRINGE THOUGHT:

Welcome from the Editors

Debbie Gochenaur and Edel Reilly

Dear Readers:

Happy 2019! We are hoping that you are having an energizing start to the new year, having had a relaxing winter break.

As your new editors of the PCTM Magazine, we would like to take this opportunity to introduce ourselves. Edel is a Professor of Mathematics at Indiana University of Pennsylvania. Edel teaches math content and methods courses for elementary and middle level pre-service teachers as well courses for the Masters in Education-Mathematics Education program at IUP and supervises student teachers. She is also the Director of Liberal Studies at IUP. Debbie is an Associate Professor of Mathematics at Shippensburg University teaching math content courses for elementary, middle, and secondary math teachers as well as methods and technology courses for secondary math teachers, supervising student teachers, and facilitating summer professional development workshops for area teachers.

A little more about Debbie: Debbie holds a B.S. in mathematics from Penn State University, an M.S. in mathematics from Shippensburg University, and a Ph.D. in mathematics education from American University. Prior to joining Shippensburg University's math department, Debbie was a high school math teacher for several years and a professor at a local private college.

A little more about Edel: Edel holds a B.S. in mathematics and economics from the National University of Ireland, an M.S. in mathematics education from UW-Madison, and a D.Ed from IUP in Curriculum and Instruction. Prior to joining IUP, Edel was a high school and middle school teacher for 15 years in Pennsylvania and Wisconsin.

FRINGE THOUGHT:

We are very excited about our first issue for you, sure that you will be able to get at least one great idea from the articles in this edition. One great idea to use in your own classroom to either use with your students or to start conversations with colleagues. Our vision for the Magazine is that it is a compilation of YOUR work! Please consider sending us your ideas for the journal - a book that you found inspiring, a technique that helped you reach a hard-to-reach student, an activity that had students truly engaged with the content. We are open to your thoughts and ideas and we are waiting to hear from you. We truly believe that this is YOUR journal.

Please email us your comments, suggestions, articles, etc. to pctm.editor@gmail.com. More information on submission guidelines may be found on page 43.

We look forward to hearing from you!

~Debbie ~Edel

Discover

JOY

in Math

#discoversumjoy

What: 2019 PCTM Conference

When: August 7th-8th

Where: Hilton Harrisburg

One North Second Street

Harrisburg, PA 17101

Who: Math teachers & math lovers

Featuring an **IGNITE SESSION** and Keynote Speakers – NCTM President Robert Q. Berry III and Dan Meyer (Chief Academic Officer at Desmos)

Go to www.pctm.org or find us on twitter @PCTMpctm for more information!

Use the Link on www.pctm.org to access the Speaker Proposal Submission site, Conference Registration Site, and Hilton Harrisburg Guest Room Reservations.

Hotel Overnight Room Reservation $141 per night, plus taxes,(Overnight Garage Parking Included) Deadline July 3, 2019, Conference Attendee Day Parking $5.00 (Walnut Street Parking Garage, adjacent to back of Hilton) Pay for day parking at the Hilton Front Desk.

FRINGE THOUGHT:

Pre-Service Teacher Day at Saint Joseph's University

Janine Firmender

On Saturday, October 20, the Teacher Education and Mathematics Departments at Saint Joseph's University hosted the Annual Preservice Mathematics Teacher Day conference. This event was also sponsored by the Pennsylvania Council for Teacher of Mathematics (PCTM) and the Pennsylvania Association for Mathematics Teacher Educators. About 100 preservice teachers from several colleges and universities in Eastern Pennsylvania attended the event.

The day began with a welcome to Saint Joseph's University from Dr. Frank Bernt, the Interim Dean for Health Services and Education, followed by a keynote speech by the past president of PCTM, Marian Avery. The keynote, entitled Teaching for Engagement and Eliciting Evidence of Student Thinking Through the use of Formative Assessment Techniques, focused on ways to use formative assessment in the classroom to determine students' levels of understanding.

Following the keynote, three breakout sessions took place. Each breakout session, consisted of 4-5 concurrent presentations that were tailored to a specific audience of preservice teachers: early childhood/elementary, middle grades, or secondary education, and general interest. In addition, two panel sessions focused on preparing for student teaching and transitioning from preservice teacher to the professional environment. Each panelist shared experiences and responded to participants' questions.

To conclude the Preservice Mathematics Teacher Day, participants reflected on their experiences and key takeaways from the event. Participants received door prizes donated by the mathematics education community.

Katie Biro, 1st Grade Teacher, Milton Area School District leads a session on implementing Number Talks

Pre-Service Teacher Day at California University of Pennsylvania

Barbara Hess and Marcia Hoover

The Western Preservice Math Teacher's Day was held on October 20, 2018 at California University of Pennsylvania. Approximately 70 participants participated in the day. The keynote speaker for the event was Dr. Laura Hummell, technology and engineering advisor for the Pennsylvania Department of Education. She delivered an interactive session on assessment. Participants were able to participate in a choice of approximately eight concurrent sessions and three afternoon workshops led by various faculty, area math teachers and experts in math education. The first poster contest was held with winners Benjamin Phillian from Indiana University of Pennsylvania and Lauren McAnany from California University of Pennsylvania as the winners. Each won a one year membership to NCTM. (Later when it was found that Lauren already belonged to NCTM, a gift of equal value was substituted.) . At the end of the day, each participant had the opportunity to select free material resources to take home thanks to generous math manipulative and textbook companies.

In Memoriam

It is with saddens that we report that **Dr. Laura Hummell** was hit in a hit and run accident while crossing the street after work in Harrisburg on November 26, 2018.
Dr. Hummell succumbed to the injuries she sustained on December 8, 2018. She is sorely missed by her former students and colleagues. Our deepest sympathies are with her family and friends at this time.

Social Justice in the Mathematics Classroom: A Critical Perspective

Ben Phillian, Yuliya Melnikova, and Janet Walker

Introduction

Teachers need creativity to show students why mathematics is an important skill to acquire, especially as young adults. By incorporating real-world events that relate math to social issues, teachers can make an impact on student mindfulness and learning, as well as motivating them.

During the Fall 2018 semester, I had an opportunity to present a topic of interest at a regional PCTM Preservice Teachers of Mathematics conference. I decided to use Gutstein and Peterson (2013) book, *Rethinking Mathematics: Teaching Social Justice by the Numbers* which uses mathematics to illustrate social justice issues. This integration helps students see the contexts and connections of real-life mathematics, which the authors describe as the "ability to read and write the world." For this article, social justice will be defined as the concept that individuals and institutions should be treated equally according to what is best for them politically, economically, and socially.

The purpose of this article is to describe an activity that uses mathematics to gain a perspective on social and environmental inequalities that communities face. Students analyze the proportions of movie theaters, community centers, and liquor stores to people living within the three-mile radius of a city or community. The activity focuses on a student's ability to work with ratios, population density, and the geometry of an urban area. The activity described below is adapted from two chapters in Gutstein and Peterson's book, as well as an article by Dr. Jason Johnson (2011) who presented this activity in their classrooms. Respectively, their work helped us construct an ideal progression of how the lesson would appear for both the teacher and their students. The original activity has been modified by adding details and restructuring the content.

Like any topic in social justice, it is imperative to understand the context of the events that occurred. Consider the case of Rodney King, an African-American living in Los Angeles in the spring of 1992. What began as a traffic stop for speeding turned into a high speed chase. Rodney King resisted arrest and was beaten with excessive force on the streets of Los Angeles by four white Los Angeles Police Department (LAPD) officers. This occurrence was captured on film from a nearby apartment and quickly became the pinpoint of attention for the media and the world. The case was taken to court, and the jury acquitted the four LAPD officers from criminal charges. Just four hours after the verdict, on April 29, 1992, riots broke out, and there was an uprising in South Central Los Angeles. Activists and civilians who were upset with the verdict flooded the streets in protest. Many small businesses were targets of arson and lootings.

Korean-Americans, Latinos, and African-Americans suffered the most from the emotional, physical, and psychological damage of the riots. To summarize the effects that the community faced, consider this excerpt from the History.com article students would be required to read: "The three days of disorder killed more than 60 people, injured almost 2,000, led to 7,000 arrests, and caused nearly $1 billion in property damage, including the burnings of more than 3,000 buildings." After a retrial one year later, two of the four LAPD officers were found guilty while the others remained acquitted; they all have left or have been fired from the LAPD since the riots.

The activity described below is designed for 7th, 8th, and/or 9th grade students and would be best to be completed over a 2-3 day period. While it touches on all eight of the Mathematical Practices (MP), this activity highlights the following three practices. First, it allows students to make sense of problems and persevere in solving them (MP #1). The epitome of this activity is to motivate students to ponder the causes of a historical event while practicing problem-solving skills. Through the modeling process, students have the opportunity to construct an awareness of the real-world in the classroom (MP #4). The use of proportions and calculated densities enhance students' grasp of depth of the injustice present. In addition, students look for and make use of the structure of the problem (MP#7). It is critical for students to understand what they are inquiring and why. Problem-solving is not always repetitive and may at times appear to be chaotic, but by having a good sense of direction and structure, students will be successful in completing the activity.

According to Pennsylvania Core Standards (https://www.pdesas.org/) this activity meets two standards significant for students' mathematical understanding. Students will analyze proportional relationships and use them to model real-world mathematical problems (CC.2.1.7.D.1). This occurs when students construct reasonable ratios of movie theaters, community centers, and liquor stores relative to a specific population. Likewise, students will determine the area of a circle in order to determine population density (CC.2.3.7.A.1). This process will be used to determine a reasonable number of resources for the South Central Los Angeles region.

Activity

On the first day, instruct small groups of students to act like city planners and consider how many supermarkets, shopping malls, movie theaters, libraries, community centers, and liquor stores they would expect to have in a city. Have the groups discuss what factors would affect their answers (number of people, size of land, etc.) and suggest considering their hometown. As a class, discuss these factors and guide the students to develop their own ratios by considering population. The students' ratios should compare a single resource to the number of people and will differ from group to group. For example:

$$\frac{1\ movie\ theatre}{14{,}000\ people} \qquad \frac{1\ community\ center}{5{,}000\ people} \qquad \frac{1\ liquor\ store}{600\ people}$$

Once groups have their ratios, have them choose a city and find its population. This could be their hometown or a nearby city and students could use recent census data. Encourage groups to choose different cities as this will aid in the discussion of population density that follows. Now students will calculate the actual number of each resource based on the population of their chosen city.

For example, if a group decided that there should be 1 movie theatre per 14,000 people, they would need to calculate the total number of movie theaters in their city using that ratio.

According to the 2010 Census, there were about 181,000 people in Tallahassee, Florida, so the calculation would be as follows:

$$\frac{14{,}000\ people}{1\ movie\ theatre} = \frac{181{,}000}{x\ movie\ theatres}$$

Based on the example above, if there are 14,000 people for every movie theatre, then students would establish that there would be 13 movie theaters in Tallahassee, Florida. Students will then complete this procedure for the number of community centers and liquor stores.

Once all groups have finished their calculations, each group will share the city or the population they chose and the number of theaters, community centers, and liquor stores they calculated based on their chosen ratios. Have each group discuss why they chose the ratios they did and how those ratios impacted their results. After all groups have shared their work, discuss population density, which is relative depending on location. Would the ratios change for a city of 2 million or for a city of less than 10,000?

At the end of the first day, assign the students to read the following two articles given below for homework: “Riots Erupt in Los Angeles” and “Los Angeles Riots by the Numbers.” Included also are some guiding questions for students to answer as homework.

1. https://www.history.com/this-day-in-history/riots-erupt-in-los-angeles

2. www.Los Angelestimes.com/local/1992riots/Los Angeles-me-riots-25-years-20170420-htmlstory.html

<u>Guiding Questions</u>

- What happened before and after the final verdict of Rodney King?
- What races were most affected by the L.A. riots?
- How did these events change the community?
- Why do you think these events occurred?
- Choose one diagram from “Los Angeles Riots by the Numbers” and reflect on it. How do these numbers make you feel?

The next day, ask students about the articles. The following questions could be used:

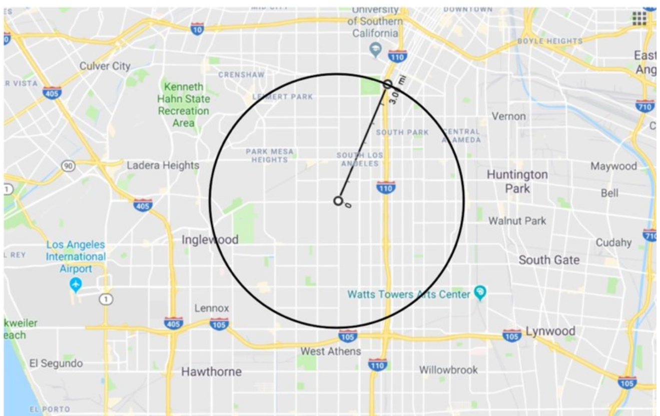
- Based on what we did the yesterday what do you think the ratios of movie theaters, community centers, and liquor stores were for South Central Los Angeles before and after the riots occurred?

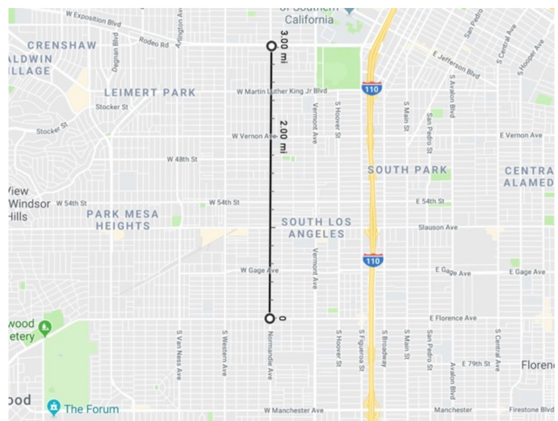
- Can you make any predictions on how these values might help us understand the cause and effect(s) of the riots?

- How can we use mathematics to explore or better understand this event?

After the riots occurred, National Public Radio (NPR) reported the actual number of movie theaters, community centers, and liquor stores in a “three-mile radius” in South Central Los Angeles with the intersection of East Florence and Normandie Avenues as the central point of the riots. Students will calculate the area of South Central Los Angeles, find the population density of that area, and then will use their group’s ratios from the previous day to determine the expected number of movie theaters, community centers, and liquor stores. At the end of the activity, the actual number of these resources in South Central Los Angeles in 1992 will be revealed to the students.

Instruct the students to work in their previous groups to find the area of South Central Los Angeles with a three-mile radius without specific directions on how to do so. Provide students with physical maps of South Central Los Angeles.





The students could determine the area in the following ways:

- Use the formula for area of a circle with a radius of three miles
  $\pi(3\ miles)^2 \approx 28.27\ square\ miles$

- Count the total number of city blocks within the circle (estimate by accounting for blocks inside and outside the circle)

- Determine the radius in city blocks.
  $Radius = 18\ blocks\ per\ mile \times 3\ miles = 54\ blocks$

- Use area formula for circle.
  $\pi(54\ blocks)^2 \approx 9{,}160\ square\ blocks$

- Create a square with a side length of 108 blocks.
  $(108\ \text{blocks})^2 \approx 11{,}664\ \text{square blocks}$

After the groups are done with their area calculations, discuss their results and methods. Include a discussion of what constitutes a square mile and/or a square block. Guide students to accept that square miles are "better" so the entire class uses the same units. During the following calculations, encourage students to label their proportions appropriately.

Students will work in their groups to find the population density of the area. State that the population in 1992 of Los Angeles was 3,544,000, and the overall size was 503 square miles. Students should compute the estimated population density as follows:

$$\frac{3{,}544{,}000\ people}{503\ square\ miles} \approx 7{,}045\ people\ per\ square\ mile$$

The following calculations use the area of 28.27 square miles found using the area of a circle method. In order to obtain the population density of this designated area, multiply the estimated density of Los Angeles (7,045 people per square mile) by the number of square miles in the area:

$$\frac{7045\ people}{1\ square\ mile} \times 28.27\ square\ miles \approx 199{,}162\ people$$

This result (191,162 people) is a rough estimate of the population within a three-mile radius of South Central Los Angeles. Now, using the ratios that the groups constructed at the beginning of the activity, students will calculate how many movie theaters, community centers, and liquor stores they would expect to find in this specific area. For example:

$$199{,}162\ people \times \frac{1\ liquor\ store}{600\ people} = 332\ liquor\ stores$$

As well as the density at which the resource appears for every one square mile:

$$\frac{332\ liquor\ stores}{28.27\ square\ miles} = 11\ liquor\ stores\ every\ square\ mile$$

Discuss how many liquor stores, community centers, and movie theaters students expected in South Central Los Angeles using their groups' ratios. Then, display the following numbers that were broadcasted by National Public Radio.

0 Movie theaters, 0 Community Centers, 640 Liquor Stores

Make it very clear to students that these numbers **<u>are not</u>** directly related to the riots. The numbers allow us to make predictions as to why this might have affected the community around them. Ask students how the statistics could potentially affect a community or future generations.

After reflecting on the actual numbers, allow students to find the density of the liquor stores within the three-mile radius. Assuming they were equally distributed across the area, how far would anyone have to walk in order to reach a liquor store? Students may compute this density in square miles and/or square blocks.

$$\frac{number\ of\ liquor\ stores}{area\ in\ square\ miles} = \frac{640\ liquor\ stores}{28.27\ sq.miles} \approx 22\ liquor\ stores\ per\ square\ mile$$

$$\frac{area\ in\ square\ blocks}{number\ of\ liquor\ stores} = \frac{9160\ square\ blocks}{640\ liquor\ stores} \approx every\ 14.3\ square\ blocks, 1\ liquor\ store$$

To put this result into perspective, display a square mile of the town in which you live and imagine if your city planners built 22 liquor stores within its bounds. Alternatively, imagine having to walk 3.8 blocks ($\sqrt{14.3}$ square blocks ≈ 3.78 blocks) in any direction and coming across a liquor store.

At the end of the activity, the students should think about the outcomes. The questions below can be discussed as a class or required for homework.

1. Based on the activity, what does it mean for something to be socially unjust? In your own words, can you define social justice?

2. How did the mathematics help you adjust your perspective on the events of Rodney King?
3. Based on the city plans your group created, why do you believe your ratios were reasonable estimates?

4. Aside from the NPR report, why do you think we solely focused on the number of movie theaters, community centers, and liquor stores found in South Central Los Angeles?

5. If you lived in South Central Los Angeles, would you do anything to help change the community?

6. Do you recognize any parallels between what happened in 1992 and what is happening today? Support your answer(s) with evidence.

<u>Conclusion</u>

Integrating social justice into mathematics helps students see the connections and contexts of real-life. Some students may have adverse attitudes toward mathematics, however, by relating math problems to social justice, we could give students to motivation to learn.

Furthermore, I believe that many of the social problems that we read about in our daily lives could be better understood if we put ourselves in another persons' shoes. Seeing the world through someone else's eyes with a critical perspective could significantly change how we handle our own decisions. Teaching the mathematics helps students develop this critical perspective, but hopefully this activity also teaches students important virtues like empathy, fairness, compassion, and respect. It is not only our job to teach mathematics, but it is our responsibility to teach students the importance of how they can make a difference in the world one person at a time.

References

Brantlinger, A. (2013). The geometry of inequality. In E. Gutstein & B. Peterson (Eds.). *Rethinking mathematics: Teaching social justice by the numbers. Milwaukee, WI: Rethinking Schools.*

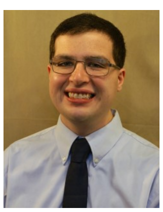
Gutstein. E. (2013). South Central Los Angeles: Ratios, density, and geometry in urban areas. In E. Gutstein & B. Peterson (Eds.). *Rethinking mathematics: Teaching social justice by the numbers. Milwaukee, WI: Rethinking Schools.*

History.com Editors. "Riots erupt in Los Angeles." *A&E Television Network. HISTORY. 3 March 2010.*

Johnson, J. (2011). Social justice lessons & mathematics. *Mathematics Teaching in the Middle School, 17(3), 174- 179.*

Kim, K., & Lauder, T. S. (2017, April 26). L.A. riots by the numbers. *Los Angeles Times. Retrieved from www.Los Angelestimes.com/local/1992riots/Los Angeles-me-riots-25-years-20170420-htmlstory.html*.

Pennsylvania Department of Education. (2014). Academic standards for mathematics. Retrieved from http://pdesas.org/Page?pageId=14.



Benjamin Phillian is an Undergraduate at Indiana University of Pennsylvania who is studying Secondary Mathematics Education. Ben is a member of the IUP Math Club and the Preservice Teachers of Mathematics (PTM) Club. In addition, he is an active participant of the collegiate and community music programs in

Dr. Yuliya Melnikova is a mathematics professor at IUP. She serves as the Developmental Mathematics Coordinator and oversees the Intermediate Algebra courses. Dr. Melnikova has presented at national and international conferences and focuses on improving the student experience in college level math classes.

Dr. Janet Walker is a Professor of Mathematics at IUP. She has also taught high school mathematics for eight years. She has presented extensively at state, regional, and national conferences and has published in several journals on topics such as using technology in the mathematics classroom, gifted education, and implementing the NCTM Standards and Mathematical Practices into the

Using Pattern Blocks to Understand Fractions and Fraction Operations

Karise Mace and Keri Stefkovich

Fractions seem to be a stumbling block for students. While many students learn how to "manage" fractions long enough to pass the test, they never really understand what fractions are or why we operate on them as we do. Naturally, this contributes to a dislike of fractions, as we tend to dislike what we do not understand. Most students can say that fractions represent a part of a whole and a few may even say that they represent parts of a set or describe them as a ratio. However, when it comes to comparing them and operating on them, most students have limited success.

The Pennsylvania Core Standards for Mathematics indicate that by the time students complete grade 5, they should be able to order fractions, understand equivalency of fractions, and add, subtract, multiply, and divide fractions (Pennsylvania Department of Education, 2014, p. 6). Anyone who teaches grades 3 – 5 knows that they teach these concepts to their students, and anyone who teaches grades 6 – 12 knows that they often must reteach these concepts to many of their students. Clearly, there is a disconnect between what is taught and what is learned when fractions are involved. It is imperative that we find a way to help our students make connections and develop fraction sense.

A few years ago, I discovered the book Exploring Mathematics: Investigations for Elementary School Teachers in which the authors introduce the idea of using pattern blocks to build student understanding of fractions and fraction operations. I was inspired by how the pattern blocks helped illuminate fractions and fraction operations and am regularly delighted to see how many light bulbs they turn on for students. In fact, each time I use pattern blocks to teach students about fractions, I hear students say "I finally understand fractions!"

In this article, we will show you how to use pattern blocks to generate deep fractional understanding in your elementary students. We know that not all fraction operations are taught in the same grade. However, we have provided tasks for each of the operations so that you will have examples of the content that is appropriate for your students.

<u>Common Misconceptions about Fractions</u>

According to research, there are four common misconceptions that students have about fractions (Melis & Goguadze, 2006).

1. Fractions behave like counting numbers.
2. All fractions are less than one.
3. Every fraction has one unique representation.
4. The numerator and denominator are independent of each other.

These misconceptions inhibit students' ability to develop a deep understanding of fractions which will facilitate their ordering of and operating on fractions. As we explore how to use the pattern blocks to work with fractions, we will take a look at how the pattern blocks can be used to dispel these misconceptions.

Getting Up Close and Personal with Pattern Blocks

Pattern blocks come in 6 different shapes. When is comes to using pattern blocks as a teaching tool for fractions only four are used: the hexagon, the trapezoid, the rhombus, and the triangle.

Before showing students how to use these blocks to represent and operate on fractions, it is important to give students time to explore the blocks and make observations about them and the relationships between them. It will not take them long to observe the following:

1. Two triangles make a rhombus.
2. Three triangles make a trapezoid.
3. Six triangles make a hexagon.
4. Two trapezoids make a hexagon.
5. Three rhombuses make a hexagon.

Once they have made these observations, they are ready to use pattern blocks to represent fractions.

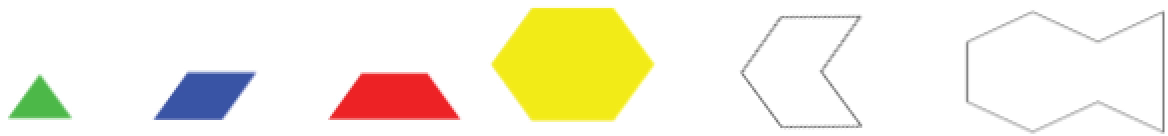
Using Pattern Blocks to Represent Fractions

Ask students to place one of the pattern blocks at the top of their desk. That block will represent "one whole". Then, discuss what fractions the other pattern blocks or combinations of pattern blocks represent. It is important to use a variety of shapes to represent "one whole," so that students will have the opportunity to see a whole in different ways. Tasks 1 and 2 allow students an opportunity to explore these concepts.

Task 1

1. Let the rhombus represent "one whole". What fractions do each of the other pattern blocks represent? (Answers: 1/2, 3. 3/2)

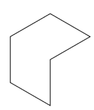
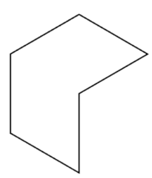
2. Now, let two hexagons represent “one whole”. What fractions do each of the following represent? (ANSWERS: $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$)



Ask students to justify their answers. As you listen to their explanations and facilitate the conversation, you will hear students begin to resolve the misconceptions that *all fractions are less than one and the numerator and denominator are independent of each other.*

Once students have gained understanding about how to represent fractions, ask them to work backwards. That is, give them a figure that represents a fraction and ask them to then represent the whole.

Task 2

1. Let the figure to the right below represent $\frac{1}{3}$.  Answer: 
   What does “one whole” look like?
2. Let the figure to the right represent $\frac{2}{3}$.  Answer:
   What does “one whole” look like?
3. Let the figure below represent $\frac{4}{9}$.  Answer: 
   What does “one whole” look like?

Again, ask students to justify their answers and encourage them to represent their answers in more than one way.

<u>Using Pattern Blocks to Compare Fractions</u>

Once students are comfortable using the pattern blocks to represent fractions, they are ready to use them to compare fractions. Students are typically successful comparing fractions with shaded models on a piece of paper when the whole is a single shape. However, if the whole is made by putting more than one shape together or if we ask students to compare numerical representations where the denominators are different, many students struggle – even when we have clearly explained the process for how and why it is necessary to get common denominators. The pattern blocks can really help build the rationale for why we need common denominators and how they are related to the original denominators of the fractions. Through building equivalent fractions in order to compare, students will realize that fractions can have multiple representations and that the numerator and denominator are dependent. Thus, the pattern blocks will aid in dispelling the misconceptions that *every fraction has a unique representation and the numerator and denominator are independent of each other.* Task 3 provides examples on how the pattern blocks can be used to compare fractions.

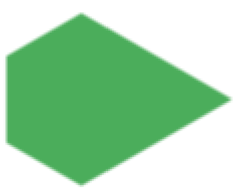
Task 3

1. Let one hexagon represent “one whole.” Is $\frac{1}{2}$ less than, greater than, or equal to $\frac{1}{3}$? Use pattern blocks to justify your answer.

(Answer:  represents $\frac{1}{2}$ and  represents $\frac{1}{3}$

so, $\frac{1}{2}$ is greater than $\frac{1}{3}$.)

2. Now, let two hexagons represent “one whole.” Is $\frac{7}{12}$ less than, greater than, or equal to $\frac{2}{3}$? Use pattern blocks to justify your answer.

(Answer:  represents $\frac{7}{12}$ and  represents $\frac{2}{3}$

so, $\frac{7}{12}$ is less than $\frac{2}{3}$.

Some students may represent the fractions using a mixture of shapes, which will still enable them to make a correct comparison. If they do this, encourage them to represent each fraction using just one shape and then to represent both fractions using the same shape. These extensions will help build the case for the need for a common denominator when comparing fractions.

<u>Using Pattern Blocks to Add and Subtract Fractions</u>

Typically, we teach students to add and subtract fractions by starting with fractions that have the same denominator. Then, we try to convince our students that you must always have a common denominator when adding and subtracting, emphasizing how to use the algorithm to do so. This seems to work fine until we start discussing how to multiply and divide fractions. If we introduce the multiplication and division algorithms before students deeply understand the algorithms for determining equivalent fractions and adding and subtracting them, they lose sight of when we need a common denominator and when we don't. All of the rules become a jumbled mess.

When we use the pattern blocks, we do not have to start with fractions that have the same denominator. In fact, if we start with different denominators, it is easier to build a case for why a common denominator is necessary. This helps provide a conceptual understanding of determining a common denominator before students begin to work on the traditional algorithm for adding and subtracting fractions. This will further resolve the misconceptions that *fractions behave like counting numbers and that the numerator and denominator are independent of each other.* Task 4 provides some examples.

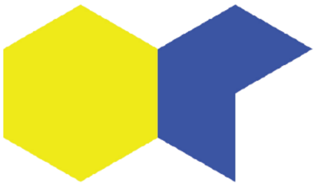
Task 4

1. Let's use one hexagon as "one whole." Determine the sum of $\frac{1}{2}$ and $\frac{1}{3}$. Students will likely model their answer with the following figure.

However, if you ask them to represent this model numerically, they will have to do a little more work. If they need the prompting, ask them to represent their answer using only one shape. Then, they should be able to provide you with the numerical answer of $\frac{5}{6}$.

2. Now, we are going to add the same fractions using two hexagons as "one whole". Determine the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

Most students will quickly be able to model their answer with the following figure.



As with the example above, students will need to model this answer using only one shape in order to determine the numerical answer of $\frac{5}{6}$.

3. Let's take a look at a subtraction exercise using one hexagon as "one whole."

We will use the same numbers we used in Task 4, Example 1, to help students make the connection between addition and subtraction. Before they model this problem, ask them to predict what the answer will be and to explain how they made their prediction.

Determine difference of $\frac{5}{6}$ and $\frac{1}{3}$.

Students will readily model $\frac{5}{6}$ using the triangles.

They may get "stuck" here, as they cannot divide 5 triangles into 3 equal groups. Give them time to struggle with this as they work toward determining the answer. If they need a little prompting, point them back to Task 4, Example 1. If they are still struggling, remind them that one hexagon represents the whole and that we are subtracting $\frac{1}{3}$ of that whole.

 represents $\frac{1}{3}$

Ask students to identify what other shape represents $\frac{1}{3}$. Students will be able to identify that two triangles also represent $\frac{1}{3}$, which makes the subtraction simple to model as they can take away those two triangles.

Ask them to write this answer numerically and then compare this answer to the one they predicted. Students will recognize that the three triangles are equivalent to one trapezoid, which is $\frac{1}{2}$ of the whole.

Provide students with a variety of addition and subtraction exercises with different combinations of blocks representing "one whole." Then, ask them what they notice about the relationship between the denominators in the original fractions and the denominator in their answer. It is also good to ask them to make observations about the numerators in the original fractions and the numerator in the answer.

After some practice using pattern blocks to add and subtract fractions, ask your students to describe how one might add and subtract fractions without the pattern blocks. If they have had time to really explore fraction addition and subtraction with pattern blocks, they will be the ones to suggest that a common denominator is a must.

Using Pattern Blocks to Multiply Fractions

Teaching students the algorithm for multiplying fractions is not quite so fraught a process as teaching them the algorithm for adding and subtracting fractions. However, there is often confusion about whether or not a common denominator is needed. Further, many students struggle to make sense of fractional multiplication because when both of the fractions are less than one, multiplication of them yields a product that is smaller than either of the factors. Because pattern blocks promote deep thinking about fractional operations, they go a long way to help students develop and retain the standard algorithm.

When we teach whole number multiplication, we talk about groups and repeated addition. Students want to apply these strategies to fractions but are often at a loss for how to do so. Discussing what we mean and how we model whole number multiplication can help students think about how they might model fraction multiplication. If 2 × 3 can be described as 2 groups of 3, then how might we describe $2\times\frac{1}{3}$? Task 5 provides examples of how to multiply fractions using pattern blocks.

TASK 5

1. Let's use one hexagon as "one whole." We will begin by multiplying a whole number by a fraction.

Determine $2\times\frac{1}{3}$.
Give students time to explore how they can use the pattern blocks to determine this product.

It is tempting to tell students right away that $2\times\frac{1}{3}$ means 2 groups *of* $\frac{1}{3}$. Don't do it! Give them time to struggle. Their struggle will help emphasize the relationship between the numerator and denominator of a fraction further dispelling a common misconception.

Once students describe $2\times\frac{1}{3}$ as 2 groups of $\frac{1}{3}$, they will quickly figure out how to model the product. Students can use this model to identify the numerical answer of $\frac{2}{3}$.

2. Now, we will use the pattern blocks to multiply two fractions using one hexagon as "one whole."

    Determine $\frac{1}{2}\times\frac{1}{3}$.

Before modeling this product, ask students to predict what the answer will be. Encourage them to look back at Example 1 and describe how this example compares to that one.

Because students are determining $\frac{1}{2}$ of $\frac{1}{3}$, students need to model $\frac{1}{3}$ first.

represents $\frac{1}{3}$ 

Now, they need to determine what half of a rhombus is. Most students will readily identify that one triangle represents $\frac{1}{2}$ of $\frac{1}{3}$. However, this is not the answer to the original exercise. Ask students to identify how much one triangle represents of the whole.

represents $\frac{1}{6}$ of the whole. 

3. Let's look at an example in which two hexagons represent "one whole".

    Determine $\frac{2}{3}\times\frac{3}{4}$

.

To model this problem using the pattern blocks, students need to begin by modeling $\frac{3}{4}$.

They should be able to quickly identify that $\frac{2}{3}$ of this model is two trapezoids.

Once the students have used the pattern blocks to model $\frac{2}{3}$ of $\frac{3}{4}$, ask them to explain how much of "one whole" this is. Students will readily see that it is $\frac{1}{2}$ of the whole. With some practice, students will be able to develop the algorithm for multiplying fractions and have a better understanding of why common denominators are unnecessary.

<u>Using Pattern Blocks to Divide Fractions</u>

While adding and subtracting fractions provides its challenges, the algorithm for dividing fractions seems arbitrary and random to many students. Why "flip" the second fraction? Why change the operation from division to multiplication?

Pattern blocks can help students understand that what we are trying to determine in fraction division is how many of one fraction is "in" another fraction. Then, we are able to build the case for why we "Keep. Change. Flip."

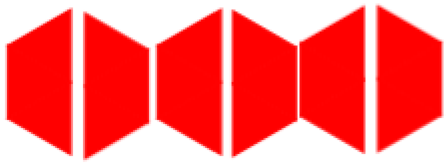
TASK 6

1. We will use one hexagon as "one whole."

Determine $3 \div \frac{1}{2}$.

Ask students to model 3 using their pattern blocks.

Then, ask students to explain what $3 \div \frac{1}{2}$ means. Once students recognize that we are trying to determine how many halves are in three wholes, ask them to model the three wholes using halves.



From this model students can easily determine that $3 \div \frac{1}{2} = 6$.

2. Now, let's look at the same exercise using two hexagons as "one whole."

Determine $3 \div \frac{1}{2}$.

Ask students to model 3 using their pattern blocks.

How many halves are there? [ANSWER: 6]

This is a good opportunity to ask your students to make observations about the dividend, divisor, and quotient. For example, students may recognize that the quotient is the product of the dividend and the denominator of the divisor. This observation lays the groundwork for the fraction division algorithm. After practicing dividing a whole number by a fraction, students are ready to divide a fraction by a fraction.

3. Let's look at an example using two hexagons as "one whole".

Determine the quotient of $\frac{1}{3}$ and $\frac{1}{2}$.

Students should begin by modeling $\frac{1}{3}$ and $\frac{1}{2}$.

 represents $\frac{1}{3}$ and represents $\frac{1}{2}$

How many halves are in $\frac{1}{3}$? Ask students to discuss their answers to this question.

They should notice that you cannot "fit" a whole half "inside" $\frac{1}{3}$. The question then becomes, how much of a half does fit in there? Because it takes three rhombuses to make $\frac{1}{2}$ of the whole and two rhombuses to make $\frac{1}{3}$ of a whole, we can say that there is $\frac{2}{3}$ of $\frac{1}{3}$ in $\frac{1}{2}$.

Be patient with division of fractions, as it may take your students some time to understand what is really happening. With practice using the pattern blocks, your students will be able to develop the algorithm for division of fractions. They are also more likely to remember it because they will understand why it works.

Fraction concepts are some of the most challenging parts of the mathematics curriculum for students in late-elementary and early middle school (Aliustaoğlu, Tuna, & Biber, 2018). Using pattern blocks in your classroom can help your students develop deep fractional understanding as they construct the standard algorithms for operating on them. Further, the pattern blocks will help resolve the four common misconceptions about fractions held by many students.

References

Aliustaoğlu, F., Tuna, A., & Biber, A. C. (2018). Misconceptions of sixth grade secondary school students on fractions. *International Electronic Journal of Elementary Education, 10*(5), 591-599.

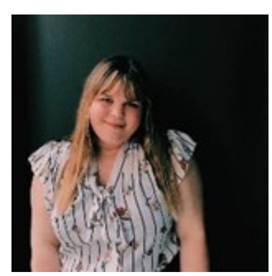
Amarasinghe, R., Burger, L., Nogin, M., Tuska, A., & Vega, O. (2013). *Exploring mathematics: investigations for elementary school teachers.* San Diego, CA: Cognella.

Melis, E., & Goguadze, G. (2006). Representation of Misconceptions. *Proceedings of the IADIS International Conference on WWW/Internet,* 208–214.

Pennsylvania Department of Education. (2014). Academic Standards for Mathematics: Numbers and operations – fractions. Retrieved from http://pdesas.org/Page?pageId=14.

Karise Mace (left) is an Instructor in the Mathematics Department at Kutztown University and former President and Vice President of the Eastern Pennsylvania Council of Teachers of Mathematics.

Keri Stefkovich (right) is a junior at Kutztown University majoring in Communication Design. She created the art for this article.



FRINGE THOUGHT:

Number Talks: How to Help Students Develop Number Sense

Valerie Long

When I taught high school mathematics, I was surprised at the lack of mathematical understanding among my students. They had trouble mentally computing what I thought were simple problems, for example $2\frac{1}{2}+1\frac{1}{2}$ or 7 x 8, or they would pull out a calculator. I believe my students had the capacity to mentally solve these problems but somewhere down the line investigating numbers in a meaningful way was lost. Many years of research has shown that traditional instruction and curriculum in the United States have given our students weak skills and a shallow understanding (Hiebert, 1999). The main cause being teaching arithmetic as a set of rules and procedures to be remembered, an algorithm (Humphreys and Parker, 2015).

The importance of algorithms should not be denied. Hyman Bass (2003) points out how arithmetic algorithms are remarkable tools because they are dependable and work with all numbers. The issue is the set of rules don't illustrate "the meaning and complexity of the steps involved" (p. 323). In the spirit of the *Common Core State Standards*, the goal of arithmetic is no longer just knowing how to compute, "rather it is the development over time of an assortment of flexible skills and procedures that are meaningfully linked to conceptual understanding" (Van de Walle, Karp, and Bay-Williams, 2019, p. 238). Number Talks are a means to help students develop this goal. This article provides a starting point for those interested in learning more about Number Talks. Specifically, this article provides a description and examples of number talks as well as resources for where to learn more.

FRINGE THOUGHT:

What are Number Talks?

Number talks are a brief daily practice (about 5 to 15 minutes) where students mentally work out a mathematics problem, typically computation, and talk about their strategies (Humphreys and Parker, 2015). Students are given a problem to solve on the board or document camera. They do not use paper and pencil for the purpose of "shifting attention from working in groups and writing to thinking by themselves" (Humphreys and Parker, 2015, p. 12). Many teachers prefer to get students out of their desks, sitting on the carpet or floor.

After students have mentally solved the problem (called a Number Talk prompt), they put up a private thumb against their chest to communicate to the teacher they have a solution. Students can also be encouraged to find other ways to solve the prompt. They can communicate these other ways by extending one or more fingers along with their thumb. Signals such as these allow students to privately communicate with the teacher without interrupting the thinking of others. When most thumbs are up, the teacher facilitates a whole class discussion of students' strategies.

The whole class discussion starts by the teacher asking for solutions to the prompt. Once all of the solutions are recorded on the board, then the teacher asks if anyone is willing to share how he or she figured out the answer. Students volunteer and first identify which solution they are defending, then explain their strategy. The teacher records, without judgment, the thinking of each volunteer on the board with equations and/or pictures. Once approaches are shared and written on the board, students are encouraged to make connections between them and adjust their reasoning. This is to connect the different strategies of students and work towards the development of flexible thinking.

The role of teachers and students are different for Number Talks. For the student, instead of being told steps to follow, now they are expected to figure something out. In addition, they are expected to explain *why* they think what they think, when knowing *how* and following steps use to be sufficient. This is a change from traditional instruction. Probably a bigger change is for us, the teachers. Most teachers were taught our role was to explain ideas clearly. What's more, this is what most of us experienced as students. As a result, Number Talks for teachers can feel very awkward at first. A teacher "must shift from being the sole authority in imparting information and confirming correct answers to assuming the interrelated roles of facilitator, questioner, listener, and learner" (Parrish, 2014, p. 12).

Two benefits of the changes in the student and teacher roles is the powerful link made between concepts and procedures and confidence in doing mathematics. In their seminal work *Making Number Talks Matter,* Humphreys and Parker (2015) point out when we start to explain how to do something, we take little bits of students' ideas and autonomy over their own thinking away. We, in essence, do the thinking for them. When we do the thinking for them it removes the emerging and often fragile authority they have over their own reasoning. Teaching in this way is a big change. These authors share ideas about how to help students move from rote thinking to thinking about what makes sense to them, namely building number sense.

Number sense is "a good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (Howden, 1989, p. 11). We want students to explore numbers meaningfully, develop multiple, flexible ways to solve problems, and connect concepts to procedures. By design, this is what Number Talks do. Here are two first graders' mental strategy for how they solved a multidigit addition problem (Berger, 2017). The prompt, 14 + 16, was written horizontally on the board and students were asked to solve it mentally.

TEACHER: (*Initially, the teacher collects values for the prompt from students*). Who has a solution and strategy he or she is willing to share?

JULIE: (*Raising her hand enthusiastically*) Ohh! Miss Smith!

TEACHER: Julie?

JULIE: I got 30.

TEACHER: Ok. How did you get 30?

JULIE: I added 10 and 10 and got 20; then I added 4 and 6 and got 10. (*Teacher is recording Julie's thinking on the board as she describes her strategy.)*

$10 + 10 = 20$

$6 + 4 = 10$

TEACHER: Okay. Everyone, do you see what she has done so far? (*Heads nod*)

JULIE: Then I added 20 and 10 and got 30.

$20 + 10 = 30$

TEACHER: Thanks Julie! Can someone explain Julie's strategy in your own words? Sean?

SEAN: She solved it by adding tens and ones. I got 30, too, but I did it different.

TEACHER: What did you do?

SEAN: I added the ones and then the tens.

TEACHER: Okay. Can you say more about that?

SEAN: I first added the 4 and 6 to get 10. Then 1 ten and another ten and got 20. Then I added 10 and 20 and got 30. (*Teacher is recording Sean's strategy on the board as he describes his strategy.)*

$4 + 6 = 10$

$10 + 10 = 20$

$10 + 20 = 30$

TEACHER: (To the class) Is anything the same or different among Julie and Sean's strategy? If so, then what?

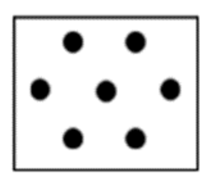
The Number Talk would continue for total time of 5 to 15 minutes with students sharing what they see as being the same or different and maybe another strategy.

<u>Types of Prompts for Number Talks.</u>

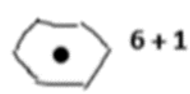
Dot cards provide a good way to begin Number Talks. The reason why is because students only have to describe what they see (Humphreys and Parker, 2015). Additionally, "arithmetic problems are emotionally loaded for many students" (p. 14) and this makes them not the best place to start Number Talks. After using dot cards for a few Number Talks, teachers can create prompts that relate to the lesson or unit they are teaching. Here are some examples of Number Talk prompts, student responses, and teacher's recording of each student's strategy.

In Example 1, teachers can tell students they will see a card with some dots on it. The teacher then explains they are to look at the card and figure out how many dots are on it without counting each one. Humphrey and Parker (2015) found it helpful to record a sketch of the students' description to match how they saw the solution. The student responses and teacher recordings are taken from their book *Making Number Talks Matter*.

Example 1: Without counting, how many dots are shown?



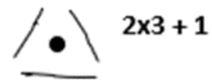
Student 1: I saw a hexagon that the dots made and then I counted the one in the middle. So, 6 and 1 is 7.



Teacher's Recording of Student 1's Strategy:

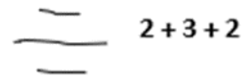
Student 2: I went by twos on the outside. I did 2 three times, which equals 6, plus the one dot in the middle.

Teacher's Recording of Student 2's Strategy:



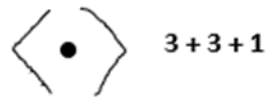
Student 3: I added rows. I added 2 plus 3 plus 2 to make 7.

Teacher's Recording of Student 3's Strategy:



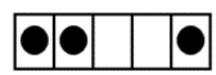
Student 4: Since it had even sides, I thought of it as a symmetry line. So three on this side and three on that side is 6, then the one in the middle is 7.

Teacher's Recording of Student 4's Strategy:



The next Number Talk prompt, Example 2, involves a Five-Frame. It is a 1 x 5 array where students are asked to figure out how many more dots are needed to make 5. Van de Walle, Karp, and Bay-Williams (2019) recommend using Five-Frames to emphasize the development of number relationships for numbers between 1 through 10. This Number Talk develops the number relationships of benchmarks of 5. Relating numbers to 5 is important because it supports student thinking about relationships with a variety of number combinations. For example, knowing 5 is two more than 3 or 2 away from 5 is 3, directly relates to basic facts like 5 = 2 + 3 or 5 – 2 = 3. What's more, connecting the spatial representation (the dots in the Five-Frame) of quantities to its verbal (five is two more than three) and symbolic representation (5 = 2 + 3) strengthens a student's ability to move between representations, which improves conceptual understanding.

Example 2: Without counting, how many more dots are needed to make 5?



Student Responses:

Student 1: I saw 2 empty spaces. There are 5 total. So 2 more are needed to make 5.

Teacher's Recording of Student 1's Strategy:

2

Student 2: There are 5 total spaces. Take off 2, that is 3. Then take off 1 more and that is 2.

Teacher's Recording of Student 2's Strategy:

5 – 2 = 3

3 – 1 = 2

Student 3. I took 1 from 5. That made 4. Then I took off the 2 dots on the right. That leaves 2.

Teacher's Recording of Student 3's Strategy:

5 – 1 = 4

Student 4: Well I know 2 dots and 1 dot are 3 dots. There are 5 total spaces with 3 dots. When I jump from 3 to 5, then that is two more.

Teacher's Recording of Student 4's Strategy:

2 + 1 = 3

3 + 2 = 5

The next couple of examples illustrate Number Talks for subtraction and multiplication prompts.

In Example 3, teachers would write the problem 63 – 28 horizontally on the board. Then ask students to solve it and show a private thumb to the chest when they are finished. This is a useful problem because with a subtrahend of 8 it encourages students to use rounding. Humphreys and Parker (2015) provided the student responses and teacher recordings listed below.

Example 3: 63 – 28. What's the difference?

Student 1: I rounded 28 to 30. Then I subtracted 30 from 63 and got 33. Then I added 2 back because I had taken away 2 too many.

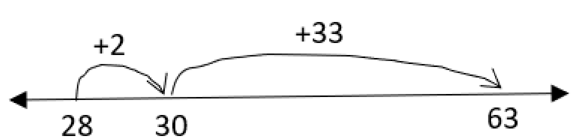
Teachers' Recording of Student 1's strategy:

63 – 30 = 33

33 + 2 = 35

Student 2: I started with 28 and added to get 30; then I added 33 and got 63. So altogether I added 2 and 33, or 35.

Teachers' Recording of Student 2's strategy:



Student 3: First I took 20 from 63 and that was 43. Then, I saw the 8 in 28 as a 3 and 5. I took away the 3 from 43 first and that was 40. Then I took away the 5 and that was 35.

Teachers' Recording of Student 3's strategy:

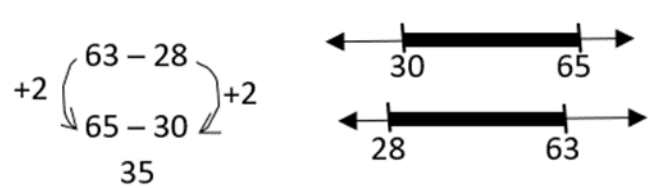
63 – 20 = 43

43 –3 = 40

40 – 5 = 35

Student 4: added 2 to 28 and got 30; then added 2 to 63 and got 65. And 65 minus 30 is 35.

Teachers' Recording of Student 4's strategy:



In Example 4, teachers would also write the problem 18 x 5 horizontally on the board. Students would then be asked to find the product. Choosing the factor of 5 may encourage students to use 10 to multiply. A factor of 18 may encourage students to round to 20 and multiply. Student responses and teacher recordings were taken from Humphreys and Parker's (2015) book *Making Number Talks Matter*.

Example 4: Find the product for 18 x 5.

Student 1: I broke the 18 into 10 and 8. First I multiplied 10 times 5 and got 50. Next I multiplied 8 times 5 and got 40. Then I added 50 and 40 and got 90.

Teachers' Recording of Student 1's strategy:

$18 \times 5 = (10 + 8) \times 5$

$10 \times 5 = 50$

$8 \times 5 = 40$

$50 + 40 = 90$

Student 2: I know 18 equals 9 times 2. First I did 2 times 5 and got 10. Then I multiplied that times 9. So, 10 x 9 is 90.

Teachers' Recording of Student 2's strategy:

$18 \times 5 = 9 \times 2 \times 5$

$2 \times 5 = 10$

$10 \times 9 = 90$

Student 3: First I did 18 times 10 and got 180. I know 5 is half of 10. So, half of 180 is 90.

Teachers' Recording of Student 3's strategy:

$18 \times 5 = 18 \times (10 \times \frac{1}{2})$

$18 \times 10 = 180$

$\frac{1}{2}$ of $180 = 90$

Student 4: I rounded 18 to 20 and I did 20 times 5 and got 100. Then I took away two fives, which is 100 minus 10. So, 90.

Teachers' Recording of Student 4's strategy:

$20 \times 5 = 100$

$2 \times 5 = 10$

$100 - 10 = 90$

Resources

There are many books and video examples available to learn more about Number Talks. As I have already mentioned, *Making Number Talks Matter* by Cathy Humphreys and Ruth Parker, creators of Number Talks, provides a wealth of information about how to enact a Number Talk and examples. Sometimes seeing how a Number Talk is carried out makes all the difference. Sherry Parrish's publications about Number Talks includes video examples.

Conclusion

Number Talks are a wonderful daily activity that helps students develop number sense and so much more. With Number Talks "teachers can provide students with some of the greatest opportunities – they can change their view of mathematics, teach them number sense, help them develop mental math skills, and at the same time, engage them in creative, open mathematics" (Humphreys and Parker, 2015, p. vii). As students engage in Number Talks they make meaning of their own mathematical ideas, connect strategies, while working towards computational fluency, a goal of the Common Core State Standards. Teachers orchestrate a discussion among students, helping them to build connections through sharing their approaches of why they solved a problem the way they did. Number Talks depend on students' sense making. Building number sense and computational fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately – using Number Talks creates students who have the potential of becoming a confident math thinker (Van de Walle, Karp, and Bay-Williams, 2019). This is game changing. To impact a student's disposition about math – I can't do math – is well worth classroom time!

References

Bass, H. (2003). Computational fluency. Algorithms and mathematical proficiency: One mathematician's perspective. *Teaching Children Mathemat*ics, 9(6): 322-327.

Berger, A. (2017). Using number talks to build procedural fluency through conceptual understanding. *Ohio Journal of School Mathematics*, 75, 1-7.

Hiebert, J. (1999). Relationship between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1): 3-19.

Howden, H. (1989). Teaching number sense. *The Arithmetic Teacher, 36*, 6-11.

Humphreys, C., & Parker, R. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding*. Portland, ME: Stenhouse.

Parrish, S. (2014). *Number talks: Helping children build mental math and computation strategies*. Sausalito, CA: Math Solutions.

Van de Walle, J., Karp, K., & Bay-Williams, J. (2019). *Elementary and middle school mathematics. Teaching Developmentally*, (10th ed.) New York, NY: Pearson.

Valerie Long is an Assistant Professor of Mathematics at Indiana University of Pennsylvania. She teaches mathematics content and methods courses for preservice and inservice teachers as well as undergraduate math classes. Her research aims to investigate teachers preferences for mathematics curriculum materials and the influencing factors. She is also interested in curricular and pedagogical issues associated with teacher and student learning, such as number talks.

2019 Pennsylvania Statistics Poster Competition

Deadline for submission:
March 1, 2019

Judging: March 2019

Announcing of Winners:
April 2019

Open to all K-12 students in Pennsylvania

Four grade level categories: K-3, 4-6, 7-9, and 10-12

Cash prizes and certificates awarded for first, second, third, and fourth places in each category

Winning posters are submitted to the national statistics poster competition

A statistics poster is a display containing graphs that summarize data, provide different points of view, and answer some question (or questions) about the data.

Ordinarily, a $96 first prize, a $72 second prize, a $48 third prize, and a $24 fourth prize will be awarded in each of the four grade level categories.

Judges will look for the following:

- Overall impact of the display for eye-catching appeal and visual attractiveness, and for its ability to draw in the viewer to investigate the graph or graphs.
- Clarity of the message's demonstration of relationships and patterns, obvious conclusions, and the ability to stand alone, even without the documentation on the back of the poster.
- Appropriateness of the graphics for the data.
- Creativity, neatness, and originality.

Registration will be open at :

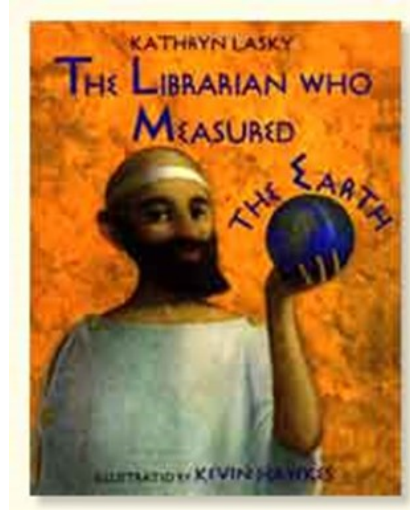
francis.edu/pa-statistics-poster-competition/

IF ANY QUESTIONS, PLEASE CONTACT:

the scienceoutreach@francis.edu or (814) 472-3878,
Beth Warner at bwarner@francis.edu or (814) 471-1215,
www.facebook.com/SFUScienceOutreachCenter, or
www.francis.edu/science-outreach-center

SAINT FRANCIS UNIVERSITY Science OUTREACH CENTER

A Math History Book Review
Laura Dick



For this semi-annual installment of my math history column, I am delighted to share one of my favorite math history children's books. *The Librarian who Measured the Earth* tells the story of Eratosthenes, a Greek scientist who traveled to Alexandria and became the tutor of King Ptolemy III's son, which eventually led him to become the head of the Alexandria Library. The illustrations in the book, by Kevin Hawkes are absolutely beautiful and serve an important function of drawing the reader into the story and eventually explaining the mathematics.

The book begins with Eratosthenes as a young child eager to ask questions about the world around him and describes Eratosthenes' love of geography. Lasky conjectures that it was this enjoyment with maps that led Eratosthenes' to seek the blessing of Ptolemy to measure the circumference of the earth. Thus the focus of the book is mathematical: just how did Eratosthenes measure the circumference? You'll have to read the book for full details, but below I'll share how I use it with my prospective students and the activity I use to go along with it.

I begin by reading the book aloud to my students, but stop on page 30, before the book explains how Eratosthenes performed his measurements. The textbook that I use has an activity manual and contains the following activity (Beckmann, 2012, p. CA-241):

The figure below shows a cross-section of the earth. At noon on June 21, the sun is directly overhead at location A, so that the sun's rays are perfectly vertical there. At the same time, 500 miles away at location B, the sun's rays make a 7.2° angle with the tip of a vertical pole (shown not to scale), which was determined by considering the shadow that the pole casts. Because the sun is far away, sun rays at the earth are (approximately) parallel. Use this information to determine the circumference of the earth, explaining your reasoning.

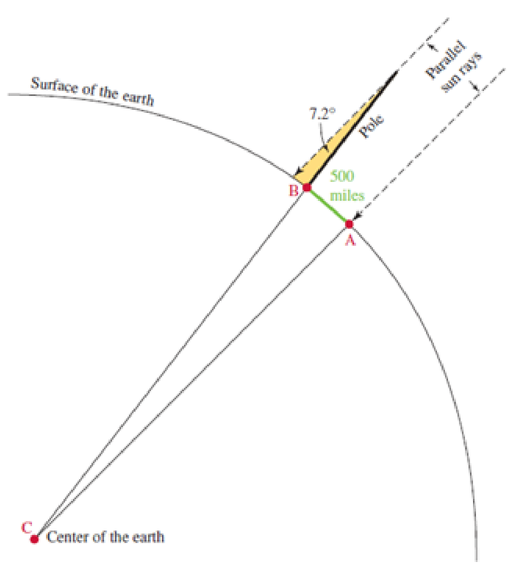


Figure 1: Figure used with Class Activity 10K (Beckmann, 2012, p. CA-241).

As you can see, in order to solve this problem, students must be familiar with geometry involving parallel lines and congruent angles. Thus, if following the PA Core standards, fits in with the 8^{th} grade math standards but my husband, a PA high school math teacher, has read the book and completed a similar activity with his high school geometry students. The task invokes the Parallel Postulate and requires the solver to think about the earth broken into sectors with a center angle, C, of 7.2°. To find the full circumference of the earth, we would need 50 sectors ($360 \div 7.2 = 50$). If the measured distance from point A to B was 500 miles, then the earth would be approximately 50*500 or 25,000 miles.

After my students complete the activity, I read the rest of the book. Having thought about the geometry of the problem before reading, I believe, helps my students make complete sense of Eratosthenes' method for determining the circumference, especially thinking about how he made his measurements of distance on land (like points A to B) and the angle measure. To measure the distance, Eratosthenes received the king's permission to use the bematists, men trained to measure distance by counting their steps; to measure the angle, Erastothenes used a gnomon (part of a sundial that deals with shadows). It is fascinating to think about the work Erastothenes put into his calculations and how accurate they were. According to National Geographic (2013), Erastothenes' method estimated the Earth to be about 28,968 miles in circumference while using modern technology, our current estimate is about 24,902 miles.

I hope those of you who teach middle and high school geometry will find this book as fun to read with your students as I do. Engaging with the history is a great way to make the math we teach more fun and relevant to our students. Enjoy!

References

Beckmann, S. (2012). *Mathematics for elementary teachers with activity manual* (4th ed.). New York, NY: Pearson.

Crooks, M. (December 16, 2013). 240 BCE: Eratosthenes Measures Circumference of Earth. Retrieved from https://www.nationalgeographic.org/thisday/jun19/eratosthenes-measures-circumference-earth/.

Lasky, K. (2008). *The Librarian who measured the earth.* Boston, MA: Little Brown Books for Young Readers.

Lara Dick is an assistant professor in the mathematics department at Bucknell University.

PCTM Honors Recent Retirees — *Thank you for your service to PCTM!*

Jane Wilburne

PCTM would like to congratulate Mary Lou Metz, Mary Ann Matras, and Tom Evitts as they retire from their positions as university mathematics educators. Mary Lou, Mary Ann, and Tom committed many hours to the organization and made significant contributions to the mission of PCTM. PCTM would like to recognize their dedication as they explore new opportunities in their retirement years.

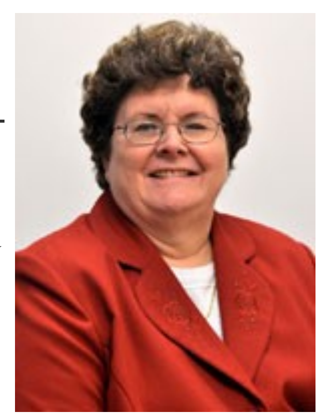
Mary Lou Metz

Mary Lou Metz, Ed.D, retired in 2017 after serving ten years as an Associate Professor of Mathematics at Indiana University of Pennsylvania. In her role at the university, she taught mathematics and pedagogical courses for elementary and middle level education majors. Mary Lou was instrumental in developing the current Masters of Education in Mathematics Education program at IUP and created and taught many courses for graduate elementary and middle-level teachers. Prior to teaching at IUP, Mary Lou taught high school mathematics for Rockwood Area High School for 27 years. She taught AP Calculus, AP Statistics, Geometry, Trigonometry, Precalculus, Algebra 2, and Algebra 1. She received the Presidential Award for Excellence in Mathematics Teaching in 1993. Mary Lou received her Ed.D. in Mathematics Education from the University of Pittsburgh in 2007.

Mary Lou was actively involved in the Pennsylvania Council of Teachers of Mathematics (PCTM). She received the PCTM's Hall of Fame Award in 2015 and the Outstanding Contribution to Mathematics Education Award in 2012. She received a $1500 grant from NCTM to implement a project through PCTM in 2007.

Mary Lou was the lead author on a data analysis and probability chapter for the Carnegie Learning's Core Math Algebra I and II, and the Geometry chapters. She published an article, "Using GAISE and NCTM standards as frameworks for teaching probability and statistics to pre-service elementary and middle school teachers" in the *Journal of Statistics Education* in 2010. She served on the editorial panel for NCTM's *Student Explorations in Mathematics* from 2009 and was appointed chair of the panel in 2011. She has conducted numerous presentations at NCTM Annual and Regional conferences, PCTM Annual Conferences, and the AMTE conference.

Mary Ann Matras



Mary Ann Matras (Ph.D) began her career at East Stroudsburg University (ESU) in 1988. She was appointed to the Mathematics Department to develop and strengthen curriculum for the teaching of mathematics at both the elementary and secondary educational levels. She designed six courses specifically tailored to the needs of K – 12 mathematics educators. She has undertaken a careful restructuring of ESU's B.S. in Mathematics-Secondary Education program to bring it in full compliance with the National Council of Teachers of Mathematics Standards. She was a major contributor to the restructuring committee for ESU's education programs to bring them in line with changes required by the Pennsylvania Department of Education. She was recognized by East Stroudsburg University with the Distinguished Professor Award. Mary Ann received her Ph.D. from the University of Maryland.

Mary Ann has been an active member of PCTM throughout her career. She served as President of the Pennsylvania Council of Teachers of Mathematics (PCTM) and as Chair of the Pennsylvania State Mathematics Coalition. She has been a member of the Executive Council of PCTM since 1989. In 2004 she received PCTM's Award for Distinguished Service and in 2014 she received the PCTM Hall of Fame Award.

Mary Ann has published more than thirty-five articles on mathematics education and related topics in journals such as the *Mathematics Teacher* and *Pennsylvania Council of Teachers of Mathematics Magazine*. She has presented more than fifty papers at regional and national conferences. She has served as an article and book reviewer for the journal *Mathematics Teacher* for over twenty years.

Thomas A Evitts

Tom Evitts (Ph.D) taught mathematics and mathematics education courses at Shippensburg University from 1999- 2017. Tom was involved in several professional development grant projects geared towards promoting the teaching and learning of mathematics across various grade levels. He co-developed a graduate course called *Algebra for Teachers* and developed online elementary mathematics content courses. Tom presented numerous seminars and workshops at the university in addition to being involved in various committees and advising roles. Prior to this, he taught secondary mathematics at South Western High School in Hanover, PA. Tom received his Ph.D. in Curriculum and Instruction from The Pennsylvania State University in 2004.

Tom has been an active contributor to the Pennsylvania Council of Teachers of Mathematics (PCTM) over his career. He presented numerous sessions at the annual PCTM conferences, worked on the program committee for several PCTM conferences, and was the co-editor of the PCTM Yearbook in 2007-2008 and 2011-2012. He received the Outstanding Contribution to Mathematics Education Award from PCTM in 2010.

Tom's contributions to national mathematics education organizations include serving as the chair of the Affiliate Connections Committee for the Association of Mathematics Teacher Educators. He has been on the audit team for the NCTM National Accreditation of Teacher Education (NCATE) Specialty Professional Association (SPA) reports for the past six years. He served as President of the Pennsylvania Association of Mathematics Teacher Educators in 2014-2106 and was President of the Central Pennsylvania Mathematics Association in 2007-2009. Tom published several refereed articles in journals such as *Mathematics Teacher*, *Pennsylvania Teacher Educator*, and the *PCTM Yearbook*. He was the editor and co-editor for the Connecting Research to Teaching department of the *Mathematics Teacher* journal for four years.

GREAT MATH TEACHING IDEAS

What are you doing with your students?

Send a picture and a short description to
pctm.editor@gmail.com

FRINGE THOUGHT:

Upcoming Conferences

Mar. 8-10, 2019 **T3 International Conference** Baltimore, MD

Mar. 23, 2019 **EPaDel Spring Section Meeting** King's College

Apr. 3-6, 2019 **NCTM Annual Meeting** San Diego, CA

Apr. 5-6, 2019 **Allegheny Mtn. Section of the MAA** Shepherd University

May 15-16, 2019 **PAMTE Symposium** Shippensburg, PA

Aug. 7-8, 2019 **PCTM Annual Conference** Harrisburg, PA

Sept. 25-27, 2019 **NCTM Regional Conference** Boston, MA

Oct. 19, 2019 **Preservice Teacher Day**
East – West Chester University

West – Penn State Behrend

Nov. 9, 2019 **EPaDel Fall Section Meeting** DeSales University

Affiliate Reports

Eastern Pennsylvania Council of Teachers of Mathematics (EPCTM)
Barb Miller

The Eastern PA Council of Teacher of Mathematics is preparing for a spring conference to be held on **April 13, 2019** at Kutztown University. The theme of the conference is "Technology with Equity". Please join us as we learn about technology and learning mathematics. The keynote speaker is Dr. Jacqueline Leonard. Dr. Leonard is a faculty member at the University of Wyoming working in Elementary and Early Childhood education. Dr. Leonard is co-author of the book *The Brilliance of Black Children in Mathematics* and author of *Culturally Specific Pedagogy in the Mathematics Classroom*.

More information will be available on our website https://epctmath.wixsite.com/epctm, follow us on Facebook (EPCTM Eastern PA Council of teachers of Mathematics) and on twitter @EPCTM. Current topics are Google Sites, Nearpod and Pear Deck (compare and contrast) and Kahoot! Are you interested in presenting a workshop and sharing how you engage your students? Please contact Dr. Mark Wolfmeyer, EPCTM president at wolfmeyer@kutztown.edu with any questions or submit your presentation/workshop topic at https://goo.gl/forms/MgZlBrHdEV4764A33 by **February 15, 2019**.

Our fall conference was an introductory workshop on Desmos offered by their professional development specialist Sean Sweeney. We had a great time learning about all the opportunities in Desmos lessons to engage students and have fun. The fall conference was on November 3, 2018 at Kutztown University and was attended by over 60 mathematics teachers and teacher candidates in the eastern Pennsylvania area.

FRINGE THOUGHT:

Mathematics Council of Western Pennsylvania (MCWP)
Yong Colen

In recent years, MCWP has supported events from the Math Game Nights to mathematics contests for secondary students. For example, the Senior High Mathematics League is a year-round contest in a team format. For the academic year 2018-2019, 22 schools and 65 teams are participating. MCWP has also sponsored the Algebra I Contest for many years. Due to the passing of the contest director, Mr. Bob Blamick, the contest is in a hiatus. More information about MCWP can be found at http://www.mcwp.us.

Pennsylvania Association of Mathematics Teacher Educators (PAMTE)

Valerie Long

PAMTE's **13th Annual Symposium** will be held May 16-17, 2019 at Shippensburg University.

PAMTE at AMTE. With AMTE is just around the corner, members of PAMTE and all friends are invited to get together for dinner on Friday night, February 8, 2019. Please let Lara Dick, lara.dick@bucknell.edu, know if you plan to attend. Lara will make reservations and let everyone know where and when to meet once she gets a headcount. Looking forward to seeing many of you there!

The third Pre-Service Teacher Day will be held on Saturday, October 19, 2019 at two different locations statewide. West Chester University will be hosting the "east" and the contacts are Dan Ilaria (DIlaria@wcupa.edu) and Brian Bowen (BBowen@wcupa.edu). Penn State Behrend will be hosing the "west" location and the contacts are Courtney Nagle (crt12@psu.edu) and Jodie Styers (jls982@psu.edu).

PAMTE is on Facebook! Our page is used to share information relevant to the mathematics education community. Check us out at https://www.facebook.com/groups/PAmatheduc/

PAMTE membership is open to all those interested in promoting and improving the education of pre-service and in-service teachers of mathematics across the Commonwealth of Pennsylvania. Our recently updated website now includes *online* membership registration. Visit http://www.pamte.org/

Pennsylvania Council of Leaders of Mathematics (PCLM)

Dave Kennedy

On March 1st, 2018, PCLM hosted a professional development mini-conference at Messiah College with the theme of "Leading Our Students to Number Sense at the Elementary Level". About 25 teachers and administrators attended. The conference program can be viewed at www.pclm.org. Thanks to Carol Buckley for all her work to schedule the site and to help create the program. This was the second consecutive academic year that PCLM hosted a successful event of this type at Messiah, with the previous year's event drawing about 40 teachers and administrators.

In August of 2018, PCLM presented a session titled "Why Math Coaches Should Know About PCLM" at the PCTM Annual Conference in Harrisburg. Many thanks to Lynn Columba for co-presenting with Dave Kennedy. The session featured information about both NCSM and PCLM.

Access to Member's Section of PCTM Website

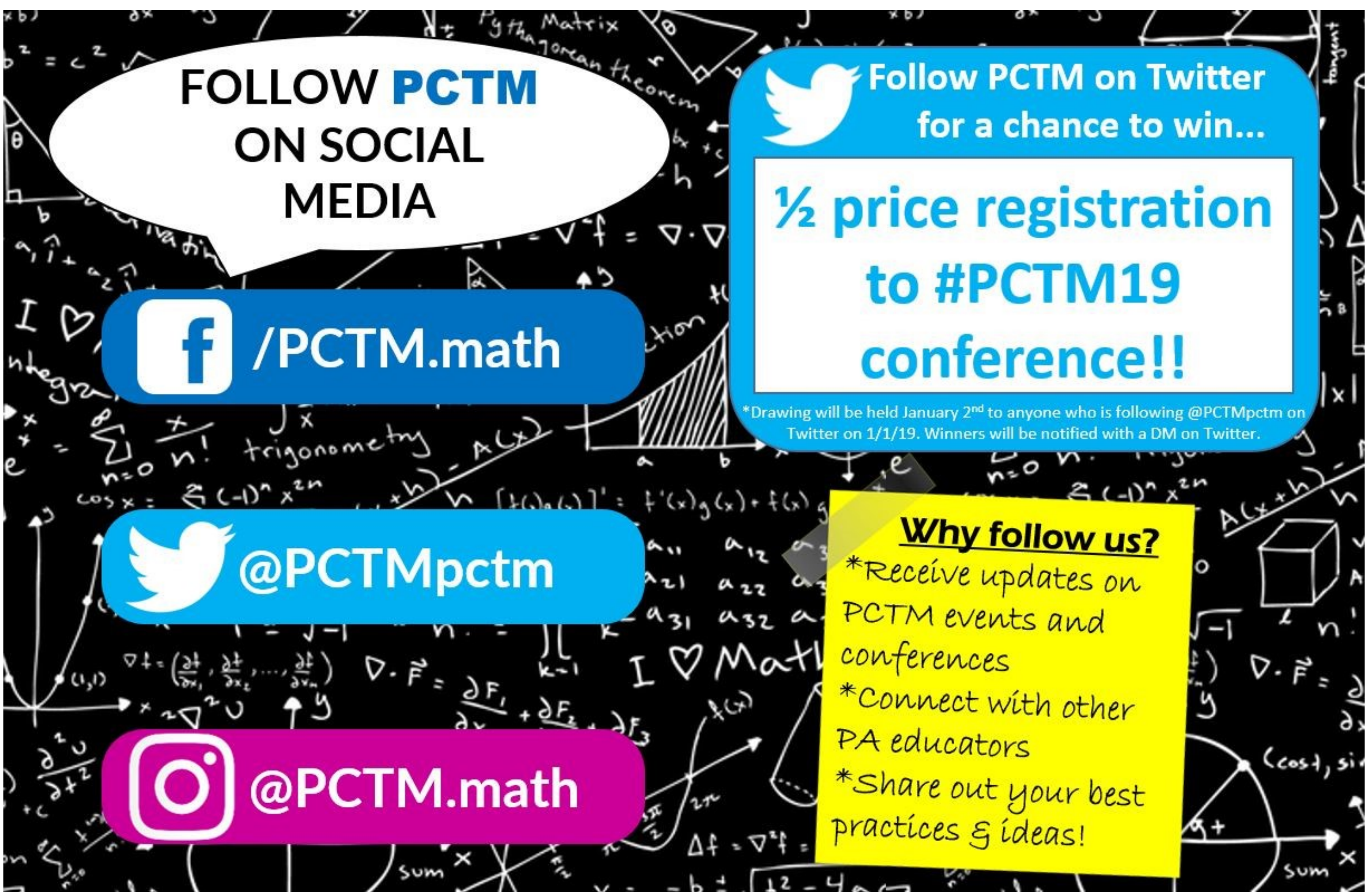
To login to the members only section:

Go to: www.pctm.org

Click on members only

username: **member**

password: **mF7eoG**



PCTM SPRING ELECTIONS

PCTM is soliciting nominations (including self-nominations) for openings on the board. The election will take place in March online.
We are seeking nominations for the following openings:

President-Elect: Per the PCTM Constitution the candidate for this position must have served as a member of the Board of Directors or as a chairperson of one of the committees listed in Article V of the Constitution, or as NCTM Representative for a period of at least two years, within a four-year period prior to nomination.

Secretary-Per the PCTM Constitution the candidate for this position must have the same qualifications as the President-Elect (See above).

***Delegate at Large*-**one position with the candidate's employment/residence from any region (Western, Central, or Eastern) in Pennsylvania

Regional Representatives-one position from each of the three areas of Pennsylvania-Western, Central, and Eastern. These positions are specific to K-12 classroom teachers.

Western-one position

Central-one position

Eastern-one position

Thank you for your nominations for the ballot. Please submit nominations to Lynn Columba hlc0@lehigh.edu by February 19, 2019.

Nominations and Election Committee

Lynn Columba, Chair

Becky Piscitella

Dave Frederickson

Call for Manuscripts—Writing in the Math Classroom

Writing can be a power teaching tool in the mathematics classroom. The benefits of using writing allows students to think and reflect about their learning. It enables discovery and communication and is accessible and a power mode for learning. We would like to include in the Spring Issue of the PCTM Magazine ways our readers have incorporated writing into their math classes. The pieces can be a full article or you are free to submit short classroom ready activities with a brief overview. We place an emphasis on classroom activities that are aligned to the Pennsylvania Core State Standards and the NCTM Principles and Standards for School Mathematics. The deadline for submission is **April 15, 2019.**

Author Guidelines:

Manuscript Format: Manuscripts should be double-spaced, with 1-inch margins on all sides, typed in 12-point font and follow the APA 6th Edition style guide. Manuscripts should be submitted in Microsoft Word. If you have a picture or graphic in the text, please include the original picture(s) in a separate file. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audience should be included in the e-mail.

Manuscript Submission: Manuscripts should be submitted electronically as an e-mail attachment to pctm.editor@gmail.com. Receipt of manuscripts will be acknowledged. After review by the editors, authors will be notified of a publication decision.

Click here for more information on registering for the conference.